Piezoelectric Beam Path Modulation for Visible Light Communication

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***Abstract*—Within the development of Beyond 5G and Sixth Generation (6G) wireless systems new communication technologies keep pushing toward high frequencies up to visible light to reduce interference and increase data rates. Many different modulation methods are available, and it is not clear yet which technique will be represented most frequently in future Visible Light Communication (VLC). As more and more demanding tasks are left to Artificial Intelligence and machines in the industry as well as in healthcare, it is essential to add further layers of resilience and security to off-the-shelf state-of-the-art VLC systems, regardless of the underlying modulation method. Therefore, in this work, two methods to modulate the physical beam path of a narrow laser beam utilizing piezoelectric mirror actuators, which can be implemented at a later stage onto existing VLC systems are proposed. While piezoelectric systems are established for specific tasks like beam stabilization in laser physics, the application in a modulation scheme for data transmission is new. The proposed concept is evaluated in the scope of upcoming wireless solutions including challenges and performance predictions. Piezoelectric beam path modulation is identified as a flexible method to enhance existing VLC systems with an additional layer of resilience and, when paired with appropriate intrusion detection mechanisms, improving security likewise.**

***Index Terms*—6G, VLC, wireless communication, laser interferometry, B5G, resilience**

I. Introduction

As the world continues to demand faster and more reliable wireless communication, research of future wireless systems is looking to new technologies to meet further demands. One technology that gained significant attention in recent years is the field of Visible Light Communication (VLC). Utilizing the spectrum of visible light to transmit data, VLC provides an alternative way of wireless communication, in previously unused frequency spaces, potentially more secure and efficient than traditional methods [1]. Within the rapid development of Beyond 5G and Sixth Generation wireless systems, VLC is one of the promised key enabling technologies [2]. Additionally, VLC is less susceptible to interference from other wireless devices and can operate in areas where Radio Frequency (RF) signals are occasionally not permitted, such as hospitals and airplanes due to interference even through walls. There is a wide range of modulation techniques and approaches used in VLC technology, such as On-Off Keying (OOK), Pulse Width Modulation (PWM) and Frequency Shift Keying (FSK) [3]. While these methods have their own advantages and disadvantages, the proposed system highlights a resilient and secure method of using a single light source for the purpose of communication, which can be activated after the failure or insecurity of the common method and is therefore independent of the initial modulation approach. While utilizing the inverse piezoelectric effect to translate voltages into the physical alteration of a narrow laser beam path [4], a simple way for data transmission is identified. The possibility to switch from a traditional modulated system to the proposed piezo-based approach, allows the implementation of an emergency communication protocol, adding resilience in case the initial modulation fails. Additionally one could also switch to the piezo approach deliberately if an eavesdropper is suspected, massively altering the modulation method and therefore hampering the intruder. These aspects of security and flexibility make this approach a highly attractive addition for applications where data privacy and security are a primary concern, such as in industrial environments with the necessity of monitoring fast-paced production lines, healthcare, or communication of sensitive information in general. Piezo-actuators are in the scope of 6G enabling technologies already candidates for modulating the channel of VLC communications but have not been included yet in the modulation scheme for data transmission itself.

The rest of the work is structured as follows: In Section II, a short summary of related work regarding VLC, modulation methods, and piezo technology is given. The principles of angular and linear beam path modulation are introduced in Section III. Section IV analyses challenges in application and performance predictions in the context of upcoming wireless networks. In Section V the findings are summarized considering the possible role in future networks.

II. Visible light communication and modulation methods

Optical Wireless Communications (OWC) and more specifically VLC as an emerging technology using the spectrum of visible light for data transmission gained significant attention in recent years due to the potential of providing high data rates, improving energy efficiency, and increasing the capacity of wireless networks by accessing new frequency bands [1]. With a wide range of applications such as indoor positioning and navigation, aspects of Internet of Things (IoT) and Industrial Internet of Things (IIoT), and high-speed data transfer, VLC can provide intrinsic security features due to the inability to penetrate walls. Known modulation methods include OOK, PWM, Color Shift Keying (CSK), Orthogonal Frequency Division Multiplexing (OFDM) and Quadrature Amplitude Modulation (QAM) [3]. Besides the specific modulation method, the emitter source is typically LED laser-based [2]. Regarding the used modulation method and emitter technology, the possible datarate and latency differ a lot as well as complexity, hardware costs, and possible security implementations. For the alteration of the channel, especially Reconfigurable Intelligent Surfaces (RISs) are a heavily discussed topic [5]. In VLC those systems could enhance range and signal strength as well as offering additional beam paths [6].

Du et al. utilized piezoelectric properties for VLC systems using LEDs to reduce the bit error rate [7]. They encode states of information by applying forces on InGaN/GaN nanostructures causing an alteration of the photoluminescence intensity output. In contrast, in this work the modulation method of the light beam itself is assumed to be disabled down to a continuous beam, modulating only the beam path. The known issue of piezo actuators' limited angle motion range is addressed by *Liu et al.* [8]. The authors presented an AlScN piezoelectric micro-electro-mechanical systems (MEMS) design with a focus on the optimization of the field of view, achieving 22.6° [8]. While their work indicates the possibility of large deflection angles in academics, for commercial systems the field of view is still very restricted [9].

Therefore in this work, the limitations of current off-the-shelf systems are considered over academic advances to realistically investigate the applicability in upcoming wireless networks.

III. Beam Path Modulation

The basic concept of Beam Path Modulation (BPM) in VLC is the encoding of information in the rapid alteration of the length or angle of a laser beam path (inherently without additional modulation of phase or amplitude), which can be detected at the receiver and therefore transmitting the information. Implementation with mechanical, moving parts is often not feasible due to the limited frequency with that mechanical components can be aligned, resulting in a low data rate, compared to conventional modulation methods [3]. In addition, even slight freedom of movement, like typical mechanical inaccuracies, can result in an unwanted shifting of the target point. Especially for large distances, high precision of the angle is crucial for hitting the detector of a receiver.

To tackle those issues, the utilization of piezoelectric actuators is proposed. These offer the possibility to convert electrical voltage directly into the modulation of a beam path by physical extension and contraction of a crystal without further movement of mechanical parts. Piezoelectric crystals, for instance, quartz (SiO_2), do not have inversion symmetry. A deformation can lead to a movement of the electrical charges linked with the atoms. As a result, a voltage accumulates at the surface when exposed to mechanical stress due to this atomic rearrangement. For VLC the inverse piezoelectric effect could be utilized. By applying an external voltage to such a piezo element, its length can be varied. This effect is mostly utilized in ultrasonic transducers for instance in medical applications [10]. For larger drive ranges, multiple piezo elements can be connected subsequently with electrodes to form a piezo stack [11]. For the concept of beam path modulation, two categories are proposed: linear and angular beam path modulation.

A. Angular Beam Path Modulation

In its simplest form, a light source directed at the receiver could encode the state "1". Turning the light source itself or a mirror in the beam path away from the receiver results in reduced incoming intensity encoding the state "0". A schematic overview is given in Fig. 1. This approach can be interpreted as

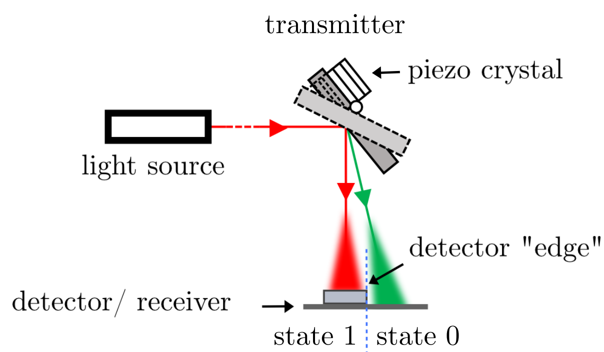

Fig. 1: Principle of angular beam path modulation: The transmitter of the information controls the angle of the piezoelectric mirror by applying voltages. At the receiver, the change of incoming power can be interpreted as different states and therefore decode the information.

OOK whereby the modulation is performed by the alteration of the beam path, not by turning the light source on and off. In contrast to traditional modulation methods, the sender is located at the reflecting mirror, while the position of the light source is not relevant: The maximum distance between the light source and the reflecting mirror is only limited by the beam divergence which implies intensity loss with increasing distance. The method is non-coherent, therefore no interference of the laser is needed. At most, unwanted self-interference can reduce the intensity at the target. As long as the contrast between the two different states "1" and "0" is large enough, the self-interference is not relevant.

B. Linear Beam Path Modulation

Another approach could be the rapid alteration of the beam path length which can represent different states and therefore transmit the information as long as the receiver can detect the different beam path lengths of the light beam. This principle has been utilized in the Michelson Interferometer introduced in [12].

In this section, the function of a Michelson Interferometer is briefly summarized and its possible application for information transport in the context of visible light communication is explained.

A laser beam is split at an optical beam splitter into two light beams, which take different paths, as shown in Fig. 2. One part of the beam (green dashed line) is reflected at a 90° angle toward mirror 1, where it is reflected back towards the beam splitter. Part of the light transmits the splitter and hits a detector (marked as the receiver in Fig. 2). Light, which initially transmits the beamsplitter (blue line) takes a different path towards mirror 2, and after reflection at the beam splitter interferes with the first beam at the detector. Interference rings become visible on the detector because of the divergence of the beam: For different positions on the detector plane, the true beam path length is affected differently by changes in one of the mirror positions. If for instance, mirror 1 is moved away from the beam splitter, the interference pattern will change and the number of rings, that shift over the detector give information about the position change of the moved mirror. For communication transport, the sender is positioned for instance at mirror 1. With a linear piezo stack, the position of the mirror can be influenced electronically, encoding the information. As a result, the electric signal is converted into a physical alteration of a beam path length. At the detector, the measured change in beam path length can be used to distinguish different states represented by different positions of mirror 1, therefore receiving the transmitted information.

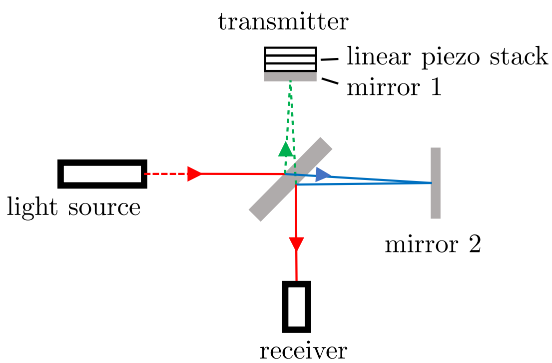

Fig. 2: Schematic illustration of a Michelson Interferometer employed for linear Beam Path Modulation. The sender alters the position of mirror 1. The change in beam path length can be detected at the receiver due to the interference with the constant beam path including mirror 2.

IV. Performance, Challenges and Use-Case

In consideration of data sheets of commercial piezo systems, the limitations of these approaches are discussed and the possible performance is estimated.

A. *Transmission range*

Suitable transmission ranges vary between angular and linear BPM.

For the linear Michelson approach, it was shown by LIGO (Laser Interferometer Gravitational-wave Observatory), which is famous for its usage in the detection of gravitation waves, one arm of a Michelson Interferometer can be up to 4 km long [13]. Especially for indoor applications, this would not be a limiting factor. However, a very expensive laser system with a long coherence length was used. The coherence length of a laser indicates over which distance interference can take place and might be significantly lower for simple, inexpensive solutions. This interference capability is necessary for the linear Michelson variant, therefore the transmission range is limited by the coherence length of the laser source. In contrast, light interference of the laser itself is an undesirable disturbance for the incoherent angular BPM: If interference stripes or rings pass over the detector when the state is switched to ”0” and thus the laser beam is directed away from the detector, this could be wrongly registered as an incoming signal.

While the linear Michelson-based BPM is limited in range, in contrast angular BPM can even have a minimum transmission range caused by the extremely small deflection angles of commercial precise high-speed piezo systems, which cover only a few millirad [9]. For example, for a transmission distance z of 5 m and a deflection angle range of 6 mrad for a state-of-the-art-system [9], the resulting x-shift on the detector plane is with $x = z \cdot tan(\alpha)$ only 3 cm. If the distance between the light source and the receiver is too small, the restricted angle range will only cause a minor displacement of the laser spot on the detector plane. As a result, the laser spot can partially remain on the detector surface during state ”0”. This situation is indicated schematically in Fig. 3, detector position C. In contrast, if the detector edge is positioned too far towards the position of state ”1”, the maximum power of this state is reduced, as indicated in Fig. 3, detector position A. In the worst case, these inaccuracies result in an insufficient change in power for a suitable Signal-to-noise ratio (SNR), as there has to be a clear distinction between the states ”0” and ”1” which are indicated by high and low measured incoming power respectively. This can be a limiting factor for detectors measuring the sum of power over their surface and cannot detect any displacements like for instance cameras. The loss in received power for inaccurate detector positioning is elaborated quantitatively for very short transmission ranges of a few meters. Note, that the detector position accuracy is influenced by the adjustment of the laser source in relation to the detector positioning. Both aspects are summarized as detector precision in the following. Furthermore, it has to be taken into account, that extremely precise piezo systems are very costly, which is why the limit for angular tolerances and ranges needs to be investigated to optimize the trade-off between precision and cost for piezo crystals in BPM. When the laser beam in angular BPM leaves the detector area during

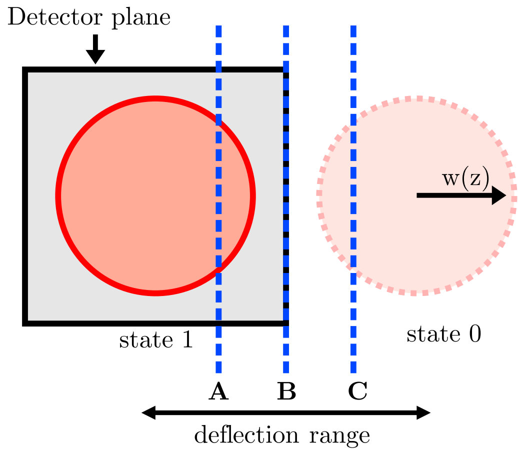

Fig. 3: Schematic representation of the two states of angular BPM corresponding to the ideal detector edge position B and two inaccurate positions A and C. The highlighted beam radius $w(z)$ indicates the position where the intensity dropped to a factor of $1/\mathrm{e}^2$ within the gaussian beam profile.

the alteration of the mirror angle as shown in Fig. 1, all power that is not hitting the detector is naturally cut off. Therefore the resulting power at the receiver can be approximated with the knife edge formula shown in eq. (1), which is typically used to measure the beamwidth of gaussian beams [14]. In this simplified approach, the shift of the laser beam over the edge of the detector surface is described by the insertion of an absorbing knife edge into the laser beam:

$$P(x) = \frac{P_0}{2}\left[1 - \operatorname{erf}\left(\frac{\sqrt{2}(x - x_d)}{w(z)}\right)\right] \tag{1}$$

P_0 is the total power of the laser beam, and x is the displacement of the laser spot in horizontal direction in the detector plane. The position of the detector edge is given with x_d. The function $w(z)$, which depends on the distance z between the laser source and the detector, describes the beam radius at which the intensity has dropped to $1/\mathrm{e}^2$ of their maximum values in the center. This beam radius is assumed to be given by a simple linear function, based on [9]: $w(z) = w_0 + d \cdot z$ w_0 describes the radius at the beam waist which is assumed as 1.5 mm, and the divergence d which is set to 1 mrad [9]. The received power in state "1" for a short transmission distance of 1 m between angular mirror and receiver in Fig. 1 is shown in Tab. I. For a maximum SNR, the ideal power for state "1" is 100 %, so that the full laser power is received.

The position of the detector edge is initially set to the respective theoretical optimum ($\Delta x_d = 0$ mm in Tab. I), which is exactly in the middle between the two states. Those two positions are at the range limits of the angular piezo mirror and therefore have the largest distance between them. The optimal detector edge position in x-direction is at $d = \frac{z}{2} \cdot tan(\alpha_{max})$, which can be derived from the simple geometry in Fig. 1, where α is the angle deviation from the central state 1. In this example the detector edge is shifted up to 6 mm from the

Tab. I: Received power in percent related to the maximum power. The detector edge is shifted away from the optimum location by the inaccuracy Δx_d in mm. With $z = \{1\,\mathrm{m}, 1.5\,\mathrm{m}, 2\,\mathrm{m}\}$ the received power is calculated for three transmission distances using (1).

Δx_d/mm	0	1	2	3	4	5	6
z							
1 m	99.8	97.7	84.1	50.0	15.9	2.3	0.0
1.5 m	100	100	98.7	90.9	67.2	32.8	9.1
2 m	100	100	100	99.2	94.5	78.8	50.0

initial spot position, simulating an inaccurate calibration and positioning. For an inaccuracy of $\Delta x_d = 0$ m, the detector edge is at the theoretical optimal position, which is shown in Fig. 3, detector position B. Note, that even for optimal detector positioning, the received power does not reach 100 % for a transmission distance of 1 m due to the gaussian intensity profile, which leads to lost power outside of the beam radius $w(z)$, which is very close to the optimal detector position. Additionally, for this short transmission distance, the received power decreases by 15.9 %. As a result, the difference between the received power of state "1" and state "0", which is at 0 % without noise is only 84.1 % of the maximum laser power. With an inaccuracy of 4 mm the difference between the two states diminishes even to 15.9 % of the maximum power, which is not feasible anymore due to expected noise from surrounding illuminating light. Therefore detector edge positioning can only have a tolerance of 2-3 mm. For larger transmission distances, this issue is less critical: At $z = 2$ m an inaccuracy of 4 mm results in a difference in received power between the two states of 94.5 % of the original power. For even larger transmission ranges due to the beam divergence, the light intensity decreases significantly and therefore the necessary detector surface to receive a significant part of the power is increasing with the distance. But in addition, the ambient noise increases for a larger detector surface, reducing the SNR (assuming a divergent beam of cost-effective laser diodes). To increase the range of laser-diode-based mirror array systems known as RIS are currently discussed for VLC systems [6].

B. Potential transmission rate of information

The data rate of BPM techniques is limited on one hand by the drive frequency of the piezo actuator and on the other hand by the maximum rate at which the intensity fluctuations can be detected at the receiver. The drive frequency of commercial angular piezo systems is typically in the range close to 10 kHz, depending on the mounted mirror size [9]. In academic approaches regarding ultrasonic motors, it was shown, that MHz frequencies can be reached but only for small sub-mm motors, which are unsuitable for the given task [15]. It has to be taken into account that commercial systems are mainly designed for precision. As long as the change in signal strength between the different states is large enough, as elaborated in the previous section, the precision of the piezo system loses relevance, especially when the transmission distance increases in case of angular BPM. Therefore, it would be beneficial to convert excess precision into a higher data rate.

This could be achieved, for instance, with a location-sensitive detector system [16]. In such a system, different positions on a detector array could represent different states, allowing transmission of more than 1 bit per mirror movement. With the example system [16] it can be estimated, that 4 bit per mirror movement could be transmitted. For continuous operation typically 80 % of the resonance frequency of the piezo is recommended, which is for the example system [9] 7.2 kHz. Therefore the data rate is expected to be $\sim$30 kbit/s. Depending on the frequency of the light source and the costs of corresponding systems, even camera systems could be feasible. However, this would require a corresponding computer-controlled position evaluation. It should be noted that, if different states are addressed quickly, there is an overshoot, which means that a single state can be approached very fast, but the piezo mirror is not immediately at rest. The overshoot can be tackled by elaborated control systems which increases the hardware cost, therefore it could be advantageous to use artificial intelligence to process the signals from the position-sensitive detector. This approach would be only feasible if there is the necessary computing capacity available at the receiver without increasing complexity and requirements.

Similarly, excess accuracy could also be used for Michelson interferometry-based linear BPM for additional transmission rate: Instead of using the full range of motion of the linear piezo system, intermediate states could be utilized. At the detector, the exact change in beam path length can be precisely detected, therefore each drive of the piezo could transmit multiple bits at once.

Additionally, a major disadvantage in the applicability of Michelson Interferometers is their susceptibility to air disturbances, which increases for larger transmission distances. In a dynamic environment like small rooms with opening doors or similar causes for noise, the bit error rate (BER) might increase to a point where linear BPM is no longer feasible. Lastly, a limiting factor for linear BPM is the detector speed, which defines the frequency with that number of interference rings, as described in section III, travel over the detector. The available detector speeds vary with the cost of the system. As other approaches, like communication with laser pulses, are likely to require photodetectors capable of GHz speeds, this limitation might also lose its relevance in the future [17].

C. Use-case, feasibility, and potential in 6G

Taking into account the performance assessment and the challenges presented in the previous section, the application of piezoelectric BPM in the context of 6G is evaluated and specific use cases are derived. The proposed method could be connected externally and at a later stage to ready-made commercial systems.

Another use case, for which additional resilience is important are healthcare environments: For stationary patients, it is often crucial to monitor vital parameters at all times. Similarly to the industrial setting, it could be switched to beam path modulation entirely if the traditional modulation method fails and only a continuous laser beam is available. This would allow the necessary information still to be transmitted. Additionally, the natural implementation of piezo mirrors in the beam path also allows (besides the possible activation of an alternative modulation method) the automated readjusting of the beam path in case of minor displacements of the receiver. Very precise and costly piezo systems perform this task already, typically in beam stabilization [18]. For the mentioned use cases the non-modulating (idle) BPM piezo system could be used for readjustments or shut down to a standby mode to reduce energy consumption.

Besides the additional layer of resilience, the traditional modulation method could also be changed to BPM intentionally, when an eavesdropper is suspected. By changing the fundamental principle of modulation to the physical beam path approach, the theft and alteration of data are impeded. Combined with anomaly detection, this process could be fully automated.

In summary, piezo-based BPM will not replace traditional modulation methods due to the lower data rate but offers an appealing option for both, additional resilience and security onto existing systems due to independence from commercial modulation systems and implementability at a later date. In comparison of the two methods discussed, angular BPM is less prone to noise than linear BPM. In addition, the requirements for the light source are lower, as angular BPM is an incoherent method, while linear BPM relies on interference.

V. Conclusion and Outlook

In this work, the concept of piezoelectric beam path modulation is introduced. While piezoelectric mirror actuators are established for beam stabilization, the full modulation for data transmission through the alteration of the physical beam path is novel. The principle was discussed in the scope of realistic application examples of future wireless networks. As the data rate is lower than conventional high-speed modulation techniques, it is not a replacement but an addition that can easily be implemented into existing systems to enhance the resilience of the communication link regardless of the original modulation method. Additionally if combined with an anomaly detection system, the proposed piezo approach increases the security of existing systems by adding the ability to change the modulation method entirely on a physical layer. Future work may investigate a full framework for fast implementation and compatibility for existing VLC systems paired with an experimental evaluation of the number of different states that can be rapidly addressed to maximize data rate.

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