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A Formal Definition for the Expressive Power of Knowledge Representation Languages

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Abstract

The notions "expressive power" or "expressiveness" of knowledge representation languages (KR-languages) can be found in most papers on knowledge representation; but these terms are usually just used in an intuitive sense. The papers contain only informal descriptions of what is meant by expressiveness. There are several reasons which speak in favour of a formal definition of expressiveness: For example, if we want to show that certain expressions in one language *cannot* be expressed in another language, we need a strict formalism which can be used in mathematical proofs.

Though we shall only consider KL-ONE-based KR-language in our motivation and in the examples, the definition of expressive power which will be given in this paper can be used for all KR-languages with model-theoretic semantics. This definition will shed a new light on the tradeoff between expressiveness of a representation language and its computational tractability. There are KR-languages with identical expressive power, but different complexity results for reasoning. Sometimes, the tradeoff lies between convenience and computational tractability. The paper contains several examples which demonstrate how the definition of expressive power can be used in positive proofs – that is, proofs where it is shown that one language can be expressed by another language – as well as for negative proofs – which show that a given language cannot be expressed by the other language.

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1. Introduction

The notions "expressive power" or "expressiveness" of knowledge representation languages (KR-languages) can be found in several papers on knowledge representation (see e.g., Woods (1983), Levesque (1986), Levesque-Brachman (1987), Nebel (1989), or Nebel-Smolka (1989)). However, the authors usually give no formal definition of expressiveness, but only informal descriptions:

> "... expressive adequacy, has to do with the expressive power of the representation – that is, what it can say. Two components of expressive adequacy are the distinctions a representation can make and the distinctions it can leave unspecified ..." (Woods (1983), p. 22)

> "The expressive power ... determines not so much what can be said, but what can be left unsaid." (Levesque-Brachman (1987), p. 82)

For many purposes such an informal view may be sufficient. However, there are at least three reasons which speak in favour of a formal approach to expressiveness. Firstly, "the need for semantics for representation languages is pretty much agreed upon" (Brachman-Levesque (1985), p. XV). Once we have a formal semantics for a language, this semantics should completely determine what the languages can express, i.e., the expressiveness of the language should only depend on this semantics. In addition, one argument for the introduction of formal semantics was that it could be used to compare the expressiveness of different KR-formalisms (see e.g. Hayes (1974), p. 5, or Nebel (1989), p. 20).[1]

A second reason is, that we need a formal definition of expressiveness if we want to prove that certain expressions in one language *cannot* be expressed in another language. For positive proofs – that is, proofs where it is shown that an expression in one language can be simulated by an expression in another language – an intuitive notion of admissible simulations is usually sufficient. But if we want to prove that a given expression cannot be simulated by *any* expression in the other language, we need a strict formalism which can be used in mathematical proofs.[2] Until now, there are just more or less informal arguments – but no mathematical proofs – for the fact that one KR-language is more expressive than another; for example:

> "... cycles add something to the expressive power. Obviously, they allow putting restrictions on semantic structures which cannot be expressed by cycle-free terminologies." (Nebel (1989), p. 146)

> "To give an impression of the expressive power of $\mathcal{U}$, we define a complex infinite structure, namely the concept mankind without rebirth. ... Note that all possible worlds satisfying the ... definition must be *infinite*. The ability to represent such an

[1]A formal definition "of change in expressive power" of a natural language is used in Keenan-Moss (1985) for comparing the expressive power of various subclasses of determiners.

[2]The situation is similar to recursive function theory, where a formal definition of computability is indespensable for proofs of non-computability, while a rather informal description of an algorithm is usually enough to demonstrate that a function is computable.

infinite structure non-recursively indicates the expressive power of $\mathcal{U}$." (Schild (1988), p. 6, 8)

"It is the case, however, that there are concepts that can be expressed in $\mathcal{FL}$ that cannot be expressed in $\mathcal{FL}^-$, such as the concept of a person with at least one son and at least one daughter: (AND (SOME (RESTRICT *child male*)) (SOME (RESTRICT *child female*))). In $\mathcal{FL}^-$ all attributes are primitive, so sons and daughters cannot play the same role (child) and yet be distinguished by their types." (Levesque-Brachman (1987), footnote 36)

We shall reconsider these three examples in Section 5. The third reason in favour of a formal definition of expressiveness is as follows. It is an empirical fact that "there is a tradeoff between expressiveness of a representation language and its computational tractability" (Levesque-Brachmann (1987), p. 78). But the connection between expressive power of a language and the complexity of reasoning over this language is not obvious from a theoretical point of view. There may be – and we shall see that there really are – KR-languages with identical expressive power but different complexity results for the reasoning component. The formal definition of expressive power in Section 3 will explain the – rather trivial – reasons for this phenomenon.[3] Sometimes, the tradeoff lies between convenience and computational tractability.

In this paper we shall only consider KR-languages based on KL-ONE (Brachman-Schmolze (1985)). But the definitions in Section 3 are formulated in a more general framework. They may be used for any other KR-language with model-theoretic semantics. Syntax and semantics of KL-ONE-based KR-languages will be introduced in Section 2.[4] I shall also mention some complexity results for subsumption determination in these languages. The formal definitions in Section 3 will be motivated by examples from this section.

Terminologies (T-boxes) can be considered as finite sets of first-order formulas. A terminological KR-language is characterized by the kind of sets of formulas that can be built. For some languages the formal semantics allows only a subclass of all first-order models as admissible models; e.g., if one wants to have features instead of roles (Nebel-Smolka (1989)) or fixed-point semantics for cyclic terminologies (Nebel (1987)). The KR-languages based on first-order predicate logic, which are defined in Section 3, will consist of a set of sets of formulas (the admissible T-boxes) and a model restriction function. A set of formulas Γ in one languages is expressed by the set of formulas Δ in the other language, if both sets have "the same" models, where equality of models is defined only w.r.t. the important predicates, i.e., w.r.t. the predicates occurring in Γ. Auxiliary predicate symbols, which may occur in Δ, will have no influence in this notion of "equality" of models.

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[3]This phenomenon can also be observed in formal language theory: The problem "$L(\alpha) = \Sigma^*$?" has quadratic complexity for regular expressions while it has exponential complexity for regular expressions with squaring. Regular expressions and regular expressions with squaring (where the regular expression $\alpha\alpha$ can be abbreviated by α^2) have the same "expressive power", they both express regular languages (see Machtey-Young (1978)).

[4]In fact, we shall only consider the T-Box formalism of these languages. The introduction will be rather short and technical. For more information and motivation see e.g. Brachman-Schmolze (1985) and Nebel (1989).

Section 3 also contains two examples and some remarks on the connection between expressive power and complexity of subsumption. In Section 4 we shall consider an alternative definition of expressive power, based on concept terms rather than on terminologies. We shall also clarify the connection between our notion of "expressiveness" and the notion "conservative extension" which is frequently used in logic. The fifth section of the paper contains examples of how the definition of expressive power can be utilized for the comparison of different KL-ONE-based KR-languages.

2. KL-ONE-based KR-Languages

In KL-ONE-based languages we start with atomic concepts and roles, and use the language formalism to define new concepts and roles. Concepts can be considered as unary predicates which are interpreted as sets of individuals whereas roles are binary predicates which are interpreted as binary relations between individuals. The languages (e.g., $\mathcal{FL}$ and $\mathcal{FL}^-$ of Levesque-Brachman (1987), $\mathcal{TF}$ and $\mathcal{NTF}$ of Nebel (1989)) differ in the kind of constructs that are allowed for the definition of concepts and roles. The next definition introduces some of the constructs used in current systems.

Definition 2.1. (concept and role terms[5])
Let **C** be a set of concept names and **R** be a set of role names. The set of *concept terms* and the set of *role terms* are inductively defined. As a starting point of the induction,
(1) any element of **C** is a concept term and any element of **R** is a role term. (atomic terms)
Now let C and D (R and S) be concept terms (role terms) already defined.
(2) Then $C \sqcap D$ is a concept term. (concept conjunction)
(3) Then $R \sqcap S$ is a role term. (role conjunction)
(4) Then $\forall R{:}C$ is a concept term. (all-in-restriction, value restriction)
(5) Then $\exists R$ is a concept term. (exists-restriction)
(6) Then $\exists R{:}C$ is a concept term. (exists-in-restriction)
(7) Let n be a non-negative integer. Then $\exists_{\geq n} R$ and $\exists_{\leq n} R$ are concept terms.
(number-restrictions)
(8) Let A be an element of **C**. Then $\neg A$ is a concept term. (atomic negation)

The language $\mathcal{FL}^-$ of Levesque-Brachman (1987) uses concept-conjunctions, all-in-restrictions and exists-restrictions. The language $\mathcal{TF}$ of Nebel (1989) uses concept-conjunctions, all-in-restrictions and number-restrictions.

The concept and role terms can be used in two different kinds of concept and role definitions: complete and incomplete definitions.

Definition 2.2. (terminologies)
Let A, B be concept names, R be a role name, D be a concept term and S be a role term.
(1) Then $A = D$ and $R = S$ are terminological axioms. (complete definition)
(2) Then $A \sqsubseteq D$ and $R \sqsubseteq S$ are terminological axioms. (incomplete definition, specialization)
(3) Then dis(A,B) is a terminological axiom. (disjointness axiom)

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[5] We do not take the usual LISP-like prefix notation, but a more succinct notation due to Schmidt-Schauß (1989).

A *terminology* (T-box) is a finite set of terminological axioms with the additional restriction that no concept or role name may appear more than once as a left hand side of a definition.

A T-box contains two different kinds of concept (role) names. *Defined concepts* (*roles*) occur on the left hand side of a complete definition. The other concepts (roles) are called *primitive concepts* (*roles*). They may be undefined or partially defined by an incomplete definition.

Let A, B be concept names and let $\mathcal{T}$ be a T-box. We say that A *directly uses* B in $\mathcal{T}$ iff B appears on the right hand side of the definition of A. Let *uses* denote the transitive closure of the relation *directly uses*. Then $\mathcal{T}$ contains a *terminological cycle* iff there exists a concept name A in $\mathcal{T}$ such that A uses A.[6] The use of terminological cycles is prohibited in almost all terminological languages.

Nebel (1989) uses complete and incomplete role and concept definitions in the T-boxes of his language $\mathcal{TF}$. In addition, he allows disjointness axioms for primitive concepts.

The following is an example of a T-box in this formalism: Let Man, Woman, Human, Male, Female and Father be concept names and let child be a role name. The T-Box T_1 consists of the following axioms:

$$\begin{aligned} \text{Man} &\sqsubseteq \text{Human} \sqcap \text{Male} \\ \text{Woman} &\sqsubseteq \text{Human} \sqcap \text{Female} \\ \text{Father} &= \text{Man} \sqcap \exists_{\geq 1}\text{child} \sqcap \forall \text{child: Human} \\ &\text{dis(Man,Woman)} \end{aligned}$$

That means a man is human and male but this is not enough to really define a man; hence the incomplete definition. A father is a man who has at least one child and who has only human children. There can be no individual who is both a man and a woman. Instead of $\exists_{\geq 1}$child we could have also used $\exists$child. Man, Woman, Human, Male and Female are primitive concepts, child is a primitive role and Father is a defined concept. This terminology does not contain a cycle.

The next definition gives a model-theoretic semantics for the languages introduced above.

Definition 2.3. (interpretations and models)
An *interpretation* I consists of a set dom(I), the domain of the interpretation, and an interpretation function which associates with each concept name A a subset A^I of dom(I) and with each role name R a binary relation R^I on dom(I), i.e., a subset of dom(I) × dom(I). The sets A^I, R^I are called extensions of A, R with respect to I.
The interpretation function – which gives an interpretation for atomic terms – can be extended to arbitrary terms as follows: Let A be a concept name, C, D be concept terms, R, S be role terms and n be a non-negative integer. Assume that C^I, D^I, R^I and S^I are already defined.

$$\begin{aligned} (C \sqcap D)^I &:= C^I \cap D^I \\ (R \sqcap S)^I &:= R^I \cap S^I \\ (\forall R{:}C)^I &:= \{ x \in \text{dom(I)};\ \text{for all } y \text{ such that } (x,y) \in R^I \text{ we have } y \in C^I \} \\ (\exists R)^I &:= \{ x \in \text{dom(I)};\ \text{there exists } y \text{ such that } (x,y) \in R^I \} \end{aligned}$$

[6]See Nebel (1989), p. 56, Definition 3.4.

$(\exists R{:}C)^I$:= $\{ x \in dom(I);$ there exists y such that $(x,y) \in R^I$ and $y \in C^I \}$
$(\exists_{\geq n} R)^I$:= $\{ x \in dom(I); card(\{ y; (x,y) \in R^I \}) \geq n \}$
$(\exists_{\leq n} R)^I$:= $\{ x \in dom(I); card(\{ y; (x,y) \in R^I \}) \leq n \}$
$(\neg A)^I$:= $\{ x \in dom(I); x \notin A^I \}$

An interpretation I is a *model* of the T-box $\mathcal{T}$ iff it satisfies

$$A^I = D^I \ (R^I = S^I) \text{ for all terminological axioms } A = D \ (R = S) \text{ in } \mathcal{T},$$
$$A^I \subseteq D^I \ (R^I \subseteq S^I) \text{ for all terminological axioms } A \sqsubseteq D \ (R \sqsubseteq S) \text{ in } \mathcal{T},$$
$$A^I \cap B^I = \emptyset \text{ for all terminological axioms dis(A,B) in } \mathcal{T}.$$

This definition shows that we may consider these languages as sublanguages of first-order predicate logic. The concept (role) names can be seen as unary (binary) predicate symbols and the concept terms as abbreviations for formulas with one free variable. For example, the term "Man $\sqcap \exists_{\geq 1}$child $\sqcap$ $\forall$child: Human" is an abbreviation for the formula

$$Man(x) \wedge \exists y \, (child(x,y)) \wedge \forall y \, (child(x,y) \rightarrow Human(y)),$$

which has the free variable x.[7] Complete definition corresponds to equivalence and incomplete definition to implication. The T-box of our example is translated into the following set of formulas:

$$\forall x \, (Man(x) \rightarrow (Human(x) \wedge Male(x)))$$
$$\forall x \, (Woman(x) \rightarrow (Human(x) \wedge Female(x)))$$
$$\forall x \, (Father(x) \leftrightarrow (Man(x) \wedge \exists y \, (child(x,y)) \wedge \forall y \, (child(x,y) \rightarrow Human(y))))$$
$$\forall x \, (\neg(Man(x) \wedge Woman(x)))$$

An important service most terminological representation systems provide is computing the subsumption hierarchy.

Definition 2.4. (subsumption of concepts and roles)
Let $\mathcal{T}$ be a T-box and let A, B (R, S) be concept names (role names).

$$A \sqsubseteq_{\mathcal{T}} B \ (R \sqsubseteq_{\mathcal{T}} S) \text{ iff } A^I \subseteq B^I \ (R^I \subseteq S^I) \text{ for all models I of } \mathcal{T}.$$

In this case we say that B *subsumes* A (S subsumes R) in $\mathcal{T}$.[8]

It is not a restriction that we have defined subsumption only for atomic terms. If we want to compare the concept terms C, D w.r.t. subsumption in $\mathcal{T}$ we may simply add concept definitions A = C and B = D to $\mathcal{T}$ and compare A and B.[9]

Since subsumption relations have to be computed very often in terminological KR-systems, it is important to know the computational complexity of this problem: How hard is it, relative to the size of the given T-box, to compute subsumption relations ? Until now, complexity results – e.g., in Levesque-Brachman (1985), Nebel (1988) or Schmidt-Schauß-

[7]Since this translation into logic follows straightforwardly from the definition of the semantics, we do not give the details for each construct. The translation of number restrictions is rather tedious and may yield very long formulas. This is one important reason for using abbreviations.

[8]Since we only allow role-conjunction as construction mechanism for roles, the computation of role subsumption is rather trivial. For this reason we shall restrict our attention to concept subsumption.

[9]A and B are meant to be concept names which don' t occur in $\mathcal{T}$.

Smolka (1988) – are usually not given w.r.t. the size of the actual T-box, but w.r.t. the size of the corresponding unfolded T-box.[10] In order to unfold a T-box we have to get rid of incomplete definitions and disjointness axioms. This will be shown for the above example. A complete description of the process can be found in Nebel (1989).

First, we *eliminate the incomplete definitions*. For any incomplete definition $A \sqsubseteq D$, a new undefined concept Rest-A is introduced which stands for the absent part of the definition of A. Applied to the example, this yields the terminology T_2

$$\begin{array}{rcl} \text{Man} & = & \text{Human} \sqcap \text{Male} \sqcap \text{Rest-Man} \\ \text{Woman} & = & \text{Human} \sqcap \text{Female} \sqcap \text{Rest-Woman} \\ \text{Father} & = & \text{Man} \sqcap \exists_{\geq 1}\text{child} \sqcap \forall\text{child: Human} \\ & & \text{dis(Rest-Man,Rest-Woman).} \end{array}$$

Now, we have only two types of concepts (roles). Primitive concepts (roles), which are completely undefined[11], and defined concepts (roles). In order to *eliminate disjointness axioms* one needs negation of primitive concepts. In the example, we obtain the terminology T_3

$$\begin{array}{rcl} \text{Man} & = & \text{Human} \sqcap \text{Male} \sqcap \text{Rest-Man} \sqcap \text{D} \\ \text{Woman} & = & \text{Human} \sqcap \text{Female} \sqcap \text{Rest-Woman} \sqcap \neg\text{D} \\ \text{Father} & = & \text{Man} \sqcap \exists_{\geq 1}\text{child} \sqcap \forall\text{child: Human.} \end{array}$$

Unfolding of a T-box means substituting defined concepts which occur on the right hand side of a definition by their defining terms. This process has to be iterated until there remain only primitive concepts on the right hand sides of the definitions. Obviously, this procedure terminates if and only if the terminology is acyclic. In the example, we finally obtain T_4:

$$\begin{array}{rcl} \text{Man} & = & \text{Human} \sqcap \text{Male} \sqcap \text{Rest-Man} \sqcap \text{D} \\ \text{Woman} & = & \text{Human} \sqcap \text{Female} \sqcap \text{Rest-Woman} \sqcap \neg\text{D} \\ \text{Father} & = & \text{Human} \sqcap \text{Male} \sqcap \text{Rest-Man} \sqcap \text{D} \sqcap \exists_{\geq 1}\text{child} \sqcap \forall\text{child: Human.} \end{array}$$

The following complexity results for concept subsumption may not be very interesting from a practical point of view (since they refer to very weak languages), but they will be important for the examples in Section 3 and 5.

Proposition 2.5. (complexity of subsumption for three small languages)
(1) The language $\mathcal{FL}^-$ is defined as follows: we allow concept-conjunction, all-in-restriction, exists-restriction and complete definition of concepts, and consider only acyclic, unfolded T-boxes. Then the complexity of the subsumption test is quadratic in the size of the T-box.
(2) If we restrict $\mathcal{FL}^-$ to $\mathcal{FL}_0$ by not allowing exists-restrictions, we get the same complexity result as for $\mathcal{FL}^-$.
(3) The language $\mathcal{TL}$ is defined as $\mathcal{FL}_0$ but we also consider acyclic T-boxes which need not be unfolded. Then the complexity of the subsumption test is co-NP-complete in the size of the T-box.

[10]See Nebel (1989a) for a discussion of this problem.
[11]In the following, "primitive" will always mean "completely undefined".

A proof for (1) can be found in Levesque-Brachman (1987)[12] and for (3) in Nebel (1989a). Nebel's co-NP-completeness result means that the problem of subsumption determination in $\mathcal{TL}$ is most likely not solvable by a polynomial time algorithm. Part (2) of the proposition is an immediate consequence of the following lemma:

Lemma 2.6. Subsumption determination in $\mathcal{FL}^-$ can be reduced in linear time to subsumption determination in $\mathcal{FL}_0$.

Proof.[13] Assume that $\mathcal{T}$ is a T-box of $\mathcal{FL}^-$. For any role R in $\mathcal{T}$ let P_R be a new primitive concept. Now substitute any $\exists$R term in $\mathcal{T}$ by P_R. This yields a T-box $\mathcal{T}_0$ of $\mathcal{FL}_0$ which has the same size as $\mathcal{T}$. In addition, we have for all concept names A, B occurring in $\mathcal{T}$ that $A \sqsubseteq_{\mathcal{T}} B$ iff $A \sqsubseteq_{\mathcal{T}_0} B$.

3. A Formal Definition of Expressive Power

In the previous section we have seen that terminologies can be considered as finite sets of first-order formulas which are built from predicate symbols without using function symbols. Different terminological KR-languages allow different sets of formulas. In Section 2, the models of such a set Γ of formulas were all the first-order models of Γ, i.e., all the interpretations which make all formulas of Γ true. In some cases – e.g., if we want to use features (see Nebel-Smolka (1989)) or if we consider fixed-point semantics for cyclic terminologies (see Nebel (1987) and Baader (1990a,1990b)) – it is necessary to take only a subclass of all first-order models as admissible models. To include features, some of the binary predicate symbols must be interpreted as partial functions, and in fixed-point semantics we do not take all fixed-points as models but only the least or the greatest. This motivates the following definition:

Definition 3.1. (KR-languages based on first-order predicate logic)
Assume that we have countably many variable symbols and countably many predicate symbols of any arity.[14] Let FO denote the set of all first-order formulas which can be built out of these symbols. A *KR1-language* (KR-languages based on first-order predicate logic) **L** consists of two parts:

(1) A subset L of the power set of FO, i.e., a set of sets of formulas.
(2) A model-restriction function Mod_L which maps a set $\Gamma \in L$ to a subclass $Mod_L(\Gamma)$ of all first-order models of Γ.

Let $\Gamma_1 \in L_1$ and $\Gamma_2 \in L_2$ for KR1-languages $\mathbf{L_1}$ and $\mathbf{L_2}$. We want to define what it means that Γ_1 is expressed by Γ_2. First, consider the example in Section 2 where T_3, an acyclic T-box without incomplete definitions and disjointness axioms, is unfolded. The unfolded T-box T_4 expresses the original T-box T_3, because T_3 and T_4 have exactly the same models. Taking this as a definition of "is expressed by" would be too restrictive, because then the T-box T_1 in Section 2 were not expressed by T_2. In models of T_2, the extensions of Rest-Man and Rest-Woman are connected with the extensions of Man and Woman, while they are absolutely free in models of T_1. Anyway, Rest-Man and Rest-Woman are only auxiliary symbols. In fact,

[12] Levesque-Brachman (1987) consider subsumption of concept terms w.r.t. the empty terminology. But it is easy to see that this is the same as concept subsumption w.r.t. unfolded terminologies as defined above.

[13] This is only a sketch of the proof. For a complete proof see Baader (1990a), Corollary 7.3.

[14] We do not restrict the definition to unary and binary predicate symbols.

we are only interested in the interpretation of the predicates which occur in T_1. For any predicate occurring in T_1 we need a corresponding predicate in T_2. In the example, the corresponding predicates bear the same name as the original ones. In general, the names of predicates should not be important, i.e., we need a translation function ψ which translates the predicate names in T_1 into the corresponding names in T_2. These ideas can be formalized as follows.

For a subset Γ of FO let Pred(Γ) denote the set of all predicate symbols occurring in Γ. Assume that we have a mapping ψ: Pred(Γ_1) $\rightarrow$ Pred(Γ_2), and models M_1, M_2 of Γ_1, Γ_2. The elements of Pred(Γ_2) which are outside of the range of ψ are auxiliary predicate symbols. We say that *M_1 is embedded in M_2 by ψ ($M_1 \subset_\psi M_2$)* iff all R in Pred(Γ_1) satisfy $R^{M_1} = \psi(R)^{M_2}$. Equality of classes of models modulo ψ-embedding is defined by extensionality, i.e., $Mod_{L_1}(\Gamma_1) =_\psi Mod_{L_2}(\Gamma_2)$ iff for all M_1 in $Mod_{L_1}(\Gamma_1)$ there exists M_2 in $Mod_{L_2}(\Gamma_2)$ such that $M_1 \subset_\psi M_2$ and for all M_2 in $Mod_{L_2}(\Gamma_2)$ there exists M_1 in $Mod_{L_1}(\Gamma_1)$ such that $M_1 \subset_\psi M_2$. We really need both directions of this definition. Obviously, any model of Γ_1 should be embeddable into a model of Γ_2. On the other hand, Γ_2 should not have more models than are needed for this purpose.[15] We are now ready for the main definition of this paper.

Definition 3.2. Let $\Gamma_1 \in L_1$ and $\Gamma_2 \in L_2$ for KR1-languages $\mathbf{L_1}$ and $\mathbf{L_2}$.
(1) Γ_1 *can be expressed* by Γ_2 iff there exists ψ: Pred(Γ_1) $\rightarrow$ Pred(Γ_2) such that $Mod_{L_1}(\Gamma_1) =_\psi Mod_{L_2}(\Gamma_2)$.
(2) $\mathbf{L_1}$ *can be expressed by* $\mathbf{L_2}$ iff for any $\Gamma_1 \in L_1$ there exists $\Gamma_2 \in L_2$ such that Γ_1 can be expressed by Γ_2 – i.e., iff there is a mapping χ: $L_1 \rightarrow L_2$ such that each Γ_1 in L_1 can be expressed by $\chi(\Gamma_1)$.
(3) $\mathbf{L_1}$ and $\mathbf{L_2}$ have *the same expressive power* iff $\mathbf{L_1}$ can be expressed by $\mathbf{L_2}$ and vice versa.

Obviously, if $\mathbf{L_1}$ is a *sublanguage* of $\mathbf{L_2}$ – i.e., $L_1 \subseteq L_2$ and, for all $\Gamma \in L_1$, $Mod_{L_1}(\Gamma) = Mod_{L_2}(\Gamma)$ – then $\mathbf{L_1}$ can be expressed by $\mathbf{L_2}$. It is easy to see that the restriction to T-boxes without incomplete definition does not change the expressive power, if the language allows conjunction. The set of all T-boxes without incomplete definition is a subset of the set of all T-boxes. For the other direction, the mapping χ is exemplified in Section 2 by the transformation of T_1 to T_2.[16] As another example, assume that we have a language which contains only acyclic T-boxes with complete definitions. Since unfolding does not change the class of models of a T-box, the restriction to unfolded T-boxes does not reduce the expressive power of this language.

In Section 2, we have introduced negation of undefined concepts in order to get rid of disjointness axioms. It was shown by an example, how a T-box with disjointness axioms can be expressed by a T-box with negation of undefined concepts.[16] But negation of undefined concepts may increase the expressive power.

Example 3.3. Let $\mathbf{L_1}$ be defined as follows: we allow concept conjunction, negation of undefined concepts, complete definition of concepts and consider only acyclic T-boxes. All first-order models of a T-box are admissible. The language $\mathbf{L_2}$ differs from $\mathbf{L_1}$ in that negation of

[15]Otherwise, a rather trivial set Γ_2 of formulas which has *all interpretations* as models could be used to express any set Γ_1 of formulas which does not contain more predicate symbols than Γ_2.

[16]See Nebel (1989), p. 62, Theorem 3.13 for a proof that the conditions of Definition 3.2 are satisfied.

undefined concepts is not allowed but disjointness axioms are allowed for undefined concepts. We have seen that $\mathbf{L_2}$ can be expressed by $\mathbf{L_1}$. The following T-box T_1 is in L_1, but cannot be expressed by a T-box in L_2: $T_1 := \{ A = P, B = \neg P, C = Q \}$, where A, B, C, P, Q are concept names (i.e., unary predicate symbols).
Assume that T_1 is expressed by the T-box $T_2 \in L_2$ w.r.t. the mapping ψ: { A, B, C, P, Q } $\rightarrow$ Pred(T_2). Without loss of generality we may assume that $\psi(A)$, $\psi(B)$, $\psi(C)$ are defined concepts with defining axioms $\psi(A) = A_1 \sqcap ... \sqcap A_k$, $\psi(B) = B_1 \sqcap .... \sqcap B_m$, $\psi(C) = C_1 \sqcap ... \sqcap C_n$, where $A_1, ..., C_n$ are undefined concepts (the case where $\psi(A)$ is undefined is similar to the case k = 1; the condition "$A_1, ..., C_n$ undefined" can be satisfied by unfolding).
(1) There do not exist i, j such that dis(C_i,C_j) is in T_2. Otherwise, the model I of T_1 defined by dom(I) := { a }, $A^I := P^I :=$ dom(I) =: Q^I =: C^I, $B^I := \emptyset$ would not have a corresponding model of T_2.
(2) We define a model J of T_2 by dom(J) := { a }, $C_1{}^J := ... := C_n{}^J :=$ dom(J). The other undefined concepts in T_2 are interpreted by the empty set.[17] Since T_1 is expressed by T_2 w.r.t. ψ there exists a model I of T_1 with $I \subset_\psi J$. We have $a \in \psi(C)^J = C^I \subseteq$ dom(I) and hence $a \in A^I$ or $a \in B^I =$ dom(I) \ A^I.
(3) Without loss of generality we may assume that $a \in A^I = \psi(A)^J = A_1{}^J \cap ... \cap A_k{}^J$. Because of the definition of J, that means that $\{ A_1, ..., A_k \} \subseteq \{ C_1, ..., C_n \}$. But then the model I' of T_1 defined by dom(I') := { a }, $B^{I'}$:= dom(I') =: $Q^{I'}$ =: $C^{I'}$, $A^{I'} := P^{I'} := \emptyset$ does not have a corresponding model of T_2. This is a contradiction to our assumption that T_1 can be expressed by a T-box T_2 of $\mathbf{L_2}$. Thus we have shown that $\mathbf{L_1}$ cannot be expressed by $\mathbf{L_2}$.

In the next example we shall see that primitive negation in Nebel's language $\mathcal{NTF}$ (Nebel (1989)) can be simulated by number-restrictions, which are also present in that language.

Example 3.4. Let $\mathbf{L_1}$ be defined as follows: we allow concept conjunction, role conjunction, all-in-restrictions, number-restrictions, negation of undefined concepts, complete definition of concepts and roles. We consider only acyclic T-boxes and take all first-order models as admissible models. The sublanguage $\mathbf{L_2}$ of $\mathbf{L_1}$ differs from $\mathbf{L_1}$ in that negation is not allowed. We shall show that $\mathbf{L_1}$ can be expressed by $\mathbf{L_2}$.
Let T_1 be a T-box of $\mathbf{L_1}$. For any undefined concept P in T_1 which occurs negated in T_1, we introduce a new role symbol R_P and a new concept symbol A_P. Now replace the unnegated occurrences of P in T_1 by $\exists_{\geq 1} R_P$ and the negated occurrences $\neg P$ by $\exists_{\leq 0} R_P$, and add a new definition $A_P = \exists_{\geq 1} R_P$. This yields a T-box T_2 of $\mathbf{L_2}$.
The translation function ψ: Pred(T_1) $\rightarrow$ Pred (T_2) is defined by $\psi(A) := A$ for all predicate symbols A in T_1 which do not occur negated in T_1 and $\psi(P) := A_P$ for all predicate symbols P which occur negated in T_1.
(1) Let I be a model of T_1. We define an interpretation J as follows: dom(J) := dom(I); for all predicate symbols A not occurring negated in T_1, $\psi(A)^J = A^J := A^I$; for all predicate symbols P occurring negated in T_1, $\psi(P)^J = A_P{}^J := P^I$ and $R_P{}^J := \{ (a,a); a \in P^I \}$.
Then $(\exists_{\geq 1} R_P)^J = P^I = A_P{}^J$ and $(\exists_{\leq 0} R_P)^J =$ dom(J) \ $P^I = (\neg P)^I$. This shows that J is a model of T_2 with $I \subset_\psi J$.

[17]For an acyclic T-box without incomplete definition, a model is completely determined by its domain and the interpretation of the primitive concepts. (1) shows that J in fact is a model.

(2) Let J be a model of T_2. We define the interpretation I by dom(I) := dom(J); for all predicate symbols A not occurring negated in T_1, $A^I := \psi(A)^J = A^J$; for all predicate symbols P occurring negated in T_1, $P^I := \{ a;$ there exists b with $(a,b) \in R_P{}^J \}$. It is easy to see that I is a model of T_1 with $I \subset_\psi J$.
Thus we have proven that $Mod_{L_1}(T_1) =_\psi Mod_{L_2}(T_2)$.

In the remainder of this section we shall consider the *connection between expressive power* of KR1-languages, as defined above, *and the complexity of the subsumption test* for the languages.[18] Let $\mathbf{L_1}$ and $\mathbf{L_2}$ be two KR1-languages such that $\mathbf{L_1}$ can be expressed by $\mathbf{L_2}$. That means that there exists a mapping χ: $L_1 \rightarrow L_2$ with the property that any $\Gamma \in L_1$ can be expressed by $\chi(\Gamma) \in L_2$. Subsumption in $\mathbf{L_1}$ can be reduced to subsumption in $\mathbf{L_2}$ as stated in the following proposition.

Proposition 3.5. Let the element Γ of L_1 be expressed by $\chi(\Gamma)$ w.r.t. ψ: Pred(Γ) $\rightarrow$ Pred($\chi(\Gamma)$). Let A, B $\in$ Pred(Γ) be predicate symbols of the same arity. Then

$$A \sqsubseteq_\Gamma B \text{ iff } \psi(A) \sqsubseteq_{\chi(\Gamma)} \psi(B).$$

Proof. Assume that $A^I \not\subseteq B^I$ for a model $I \in Mod_{L_1}(\Gamma)$, i.e., $A \not\sqsubseteq_\Gamma B$. There exists a model J $\in Mod_{L_2}(\chi(\Gamma))$ such that $I \subset_\psi J$. Then $\psi(A)^J = A^I \not\subseteq B^I = \psi(B)^J$ which shows $\psi(A) \not\sqsubseteq_{\chi(\Gamma)} \psi(B)$. The other direction can be shown in the same way.

But the reduction function χ need not be polynomial, i.e., the size of $\chi(\Gamma)$ may be exponential – or even worse – in the size of Γ. The function χ may even be non-computable. If we keep this in mind, it is not surprising that two KR1-languages may have the same expressive power but different complexity – or even computability – behaviour with respect to subsumption or other kinds of reasoning. We have already seen such an example. The languages $\mathcal{FL}_0$ and $\mathcal{TL}$, as defined in Proposition 2.5, have the same expressive power since unfolding does not change the class of models. But the complexity of subsumption determination in $\mathcal{FL}_0$ is quadratic while it is co-NP-complete in $\mathcal{TL}$.

It is not enough to consider complexity results for the languages. The compactness with which the language can express things is also very important. For example, it makes no sense to express number-restrictions such as $\exists_{\geq 1000}R$ by horribly large first-order formulas (in this case with 1000 variables) though the complexity of subsumption w.r.t. the size of the T-box containing the number-restrictions may be worse than w.r.t. the size of the set of first-order formulas.

For some purposes a strengthened version of Definition 3.2 – which might require that the mapping χ: $L_1 \rightarrow L_2$ is computable or that it is computable in polynomial time etc. – may be more appropriate; again a notion familiar from computability theory.

[18] Subsumption for KR1-languages is defined in the obvious way. Let **L** be such a language and let $\Gamma \in$ L. For A, B $\in$ Pred(Γ) with the same arity we define: $A \sqsubseteq_\Gamma B$ iff $A^I \subseteq B^I$ for all $I \in Mod_L(\Gamma)$.

4. Alternative Definitions of Expressive Power

In the first part of this section we shall compare our definition of expressive power with the notion "conservative extension of a first-order theory", a concept frequently used in logic (see e.g., Andrews (1986)). In the second part we shall consider a special class of KR1-languages where the sets of formulas are given by acyclic T-boxes.

4.1. Conservative Extensions

Let $\mathbf{L_1}$ be a KR1-language, let Γ_1 be an element of L_1, and let P, Q be predicate symbols of arity n occurring in Γ_1. We have defined $P \sqsubseteq_{\Gamma_1} Q$ iff $P^I \subseteq Q^I$ for all models $I \in Mod_{L_1}(\Gamma_1)$. Consider the formula $S(P,Q) := \forall x_1 \forall x_2 \ldots \forall x_n \; (P(x_1,x_2,\ldots,x_n) \rightarrow Q(x_1,x_2,\ldots,x_n))$. Obviously, $P^I \subseteq Q^I$ for all models $I \in Mod_{L_1}(\Gamma_1)$ iff S(P,Q) is valid in all models $I \in Mod_{L_1}(\Gamma_1)$.

Proposition 3.5 can now be reformulated as follows: Assume that Γ_1 is expressed by Γ_2 w.r.t. ψ, where $\Gamma_2 \in L_2$ for a KR1-language $\mathbf{L_2}$. Let P, Q $\in$ Pred(Γ_1) be predicate symbols of the same arity. Then S(P,Q) is valid in all models $I \in Mod_{L_1}(\Gamma_1)$ iff $S(\psi(P),\psi(Q))$ is valid in all models $J \in Mod_{L_2}(\Gamma_2)$.

One may now ask whether a similary property holds for arbitrary formulas, and not only for formulas of the form S(P,Q). This is related to the question whether Γ_2 is a conservative extension of Γ_1.

Let Γ be a set of first-order formulas, and let A be a first-order formula. The language $\mathcal{L}(\Gamma)$ of Γ is the set of all predicate, function, and constant symbols occurring in Γ. The formula A is a formula over $\mathcal{L}(\Gamma)$ iff it only contains predicate, function, and constant symbols from $\mathcal{L}(\Gamma)$. We say that Γ (semantically) implies A ($\Gamma \models A$) iff each model of Γ is a model of A.

Definition 4.1. Let Γ_1, Γ_2 be sets of first-order formulas such that $\mathcal{L}(\Gamma_1) \subseteq \mathcal{L}(\Gamma_2)$. Then Γ_2 is a *conservative extension* of Γ_1 iff all formulas A over $\mathcal{L}(\Gamma_1)$ satisfy $\Gamma_1 \models A$ iff $\Gamma_2 \models A$.

Let $\mathbf{L_1}$ and $\mathbf{L_2}$ be KR1-languages such that $\mathbf{L_1}$ can be expressed by $\mathbf{L_2}$, and let $\Gamma_1 \in L_1$ be expressed by $\Gamma_2 \in L_2$ w.r.t. ψ. Because of the translation function ψ, not even the precondition $\mathcal{L}(\Gamma_1) \subseteq \mathcal{L}(\Gamma_2)$ in the definition of conservative extension needs to be satisfied. In order to cope with this problem we shall consider an obvious modification of Definition 4.1.

Definition 4.2. Let Γ_1, Γ_2 be sets of first-order formulas, and let ψ: $\mathcal{L}(\Gamma_1) \rightarrow \mathcal{L}(\Gamma_2)$ be a mapping such that, for each n-ary predicate symbol P (resp. n-ary function symbol f, constant symbol c) in $\mathcal{L}(\Gamma_1)$, $\psi(P)$ (resp. $\psi(f)$, $\psi(c)$) is an n-ary predicate symbol (resp. n-ary function symbol, constant symbol). For a formula A over $\mathcal{L}(\Gamma_1)$, let $\psi(A)$ be the formula over $\mathcal{L}(\Gamma_2)$ which is obtained from A by replacing the symbols of $\mathcal{L}(\Gamma_1)$ by their ψ-images.
We say that Γ_2 is a *conservative extension* of Γ_1 *w.r.t.* ψ iff all formulas A over $\mathcal{L}(\Gamma_1)$ satisfy $\Gamma_1 \models A$ iff $\Gamma_2 \models \psi(A)$.

Because "$\Gamma \models A$" means that *all* models of Γ are models of A, we can only hope to get a connection between the notions "$\Gamma_1 \in L_1$ is expressed by $\Gamma_2 \in L_2$ w.r.t. ψ" and "Γ_1 is a conservative extension of Γ_2 w.r.t. ψ" if we restrict our attention to KR1-languages without model restriction. A KR1-language **L** is a KR1-language without model restriction iff for all Γ

in L, $Mod_L(\Gamma)$ is the class of all first-order models of Γ. But even in this case, the fact that $\Gamma_1 \in L_1$ is expressed by $\Gamma_2 \in L_2$ w.r.t. ψ does not imply that Γ_2 is a conservative extension of Γ_1 w.r.t. ψ.

Example 4.3. Let $\mathbf{L_1}$ and $\mathbf{L_2}$ be KR1-languages without model restriction such that $\Gamma_1 := \{ \exists x\ P(x) \} \in L_1$ and $\Gamma_2 := \{ \exists x\ P(x), \exists y\ \neg P(y) \} \in L_2$. It is easy to see that Γ_1 is expressed by Γ_2 w.r.t. $\psi = id$. However, Γ_2 is *not* a conservative extension of Γ_1 w.r.t. ψ; for example, $A = \exists y\ \neg P(y)$ is a formula over $\mathcal{L}(\Gamma_1)$ and $\Gamma_2 \models \psi(A) = A$, but the interpretation M which has $dom(M) = P^M = \{ a \}$ is a model of Γ_1 which is not a model of A.

The problem stems from the fact that, in the definition of "M_1 is embedded in M_2 by ψ", we did not require that $dom(M_1) = dom(M_2)$. In the case of KL-ONE-based KR-languages, this seems to be quite correct since we are only interested in the extensions of the concepts and roles defined in our T-boxes; we are usually not interested in additional individuals which may, or may not be in the domain.

Lemma 4.4. Let ψ: $Pred(\Gamma_1) \to Pred(\Gamma_2)$ be a mapping, let A be a formula over $Pred(\Gamma_1)$, and let M_1, M_2 be interpretations such that $M_1 \subset_\psi M_2$ and $dom(M_1) = dom(M_2)$. Then M_1 is a model of A iff M_2 is a model of $\psi(A)$.

If $M_1 \subset_\psi M_2$ and $dom(M_1) = dom(M_2)$, we say that M_1 is embedded in M_2 by ψ with equality of domains.

Proposition 4.5. Let $\mathbf{L_1}$ and $\mathbf{L_2}$ be KR1-languages without model restriction. Assume that $\Gamma_1 \in L_1$ is expressed by $\Gamma_2 \in L_2$ w.r.t. ψ such that all embeddings of models are embeddings with equality of domains. Then Γ_2 is a conservative extension of Γ_1 w.r.t. ψ.

Proof. Let A be a formula over $Pred(\Gamma_1)$. We have to show that $\Gamma_1 \models A$ iff $\Gamma_2 \models \psi(A)$.

(1) Assume that $\Gamma_1 \models A$ is not satisfied, i.e., there exists a model M_1 of Γ_1 such that M_1 is not a model of A. The assumption of the proposition implies that there exists a model M_2 of Γ_2 such that $M_1 \subset_\psi M_2$ and $dom(M_1) = dom(M_2)$. By Lemma 4.4, M_2 is not a model of $\psi(A)$.

(2) The other direction can be proved analogously.

4.2. An Alternative Definition for a Special Case

In Section 2 we have seen that acyclic T-boxes without incomplete definitions and disjointness axioms can be unfolded. In this case we may restrict ourselves to unfolded T-boxes without changing the expressive power. The set of all admissible T-boxes is then completely determined by the set of all admissible concept and role terms. In addition, subsumption of concepts w.r.t. a terminology can be reduced to subsumption between concept terms.[19] This motivates the following definition, which defines expressive power only w.r.t. concept and role terms.

Definition 4.6. Let $\mathbf{F_1}$ and $\mathbf{F_2}$ be two sets of concept and role terms.[20]

(1) We say that $A \in \mathbf{F_1}$ is *equivalent* to $B \in \mathbf{F_2}$ ($A \equiv B$) iff $\{ A^I$; where I is an interpretation $\} = \{ B^J$; where J is an interpretation $\}$.

[19] See Nebel (1989), Section 3.2.5.

[20] We shall always assume that these sets of terms contain all atomic concept and role terms. The sets differ in the way more complex terms can be built.

(2) $\mathbf{F_1}$ *can be expressed by* $\mathbf{F_2}$ iff for all $A \in \mathbf{F_1}$ there exists $B \in \mathbf{F_2}$ such that $A \equiv B$.
(3) $\mathbf{F_1}$ and $\mathbf{F_2}$ have *the same expressive power* iff $\mathbf{F_1}$ can be expressed by $\mathbf{F_2}$ and vice versa.

The following is an example of two sets of concept terms which have the same expressive power, where expressive power is meant in the sense of Definition 4.6.

Example 4.7. The set $\mathbf{F_1}$ is defined as follows: a concept term is a finite conjunction of negated and unnegated concept names (e.g., $A \sqcap \neg B \sqcap C$ is an admissible concept term). In $\mathbf{F_2}$ we only allow conjunction of unnegated concept names but we have an additional term $\bot$, which has to be interpreted as the empty set.
(1) $\mathbf{F_2}$ can be expressed by $\mathbf{F_1}$ since the term $\bot$ is equivalent to the term $A \sqcap \neg A$.
(2) $\mathbf{F_1}$ can be expressed by $\mathbf{F_2}$. This can be seen in the following way. Let the concept term D be an element of $\mathbf{F_1}$. If D contains A and $\neg A$ for some concept name A, then D is equivalent to the term $\bot$. Otherwise, each literal $\neg A$ may be replaced by a new concept name A'. This yields a concept term $D' \in \mathbf{F_2}$ which is equivalent to the original term D.

We shall now investigate the connection between this definition of expressive power and the definition given in Section 3.

Definition 4.8. Let $\mathbf{F}$ be a set of concept and role terms.
The KR1-language $\mathbf{L(F)}$ is defined as follows:
(1) L(F) contains all finite T-boxes of the form $T = \{ ..., A = D, ... \}$, where A is a concept (role) name not occurring anywhere else in T[21] and $D \in \mathbf{F}$ is a concept (role) term.
(2) $Mod_{L(F)}(T)$ is the class of all first-order models of T.

If $\mathbf{L(F_1)}$ can be expressed by $\mathbf{L(F_2)}$ (according to Definition 3.2) then $\mathbf{F_1}$ can be expressed by $\mathbf{F_2}$ (according to Definition 4.6),[22] but not necessarily vice versa.

Example 4.9. Let $\mathbf{F_1}$ and $\mathbf{F_2}$ be as in Example 4.7. We have already seen that $\mathbf{F_1}$ can be expressed by $\mathbf{F_2}$ (according to Definition 4.6). We shall now show that $\mathbf{L(F_1)}$ cannot be expressed by $\mathbf{L(F_2)}$ (according to Definition 3.2).
Consider the T-box $T_1 := \{ A = P, B = \neg P \} \in \mathbf{L(F_1)}$. Assume that T_2 is a T-box of $\mathbf{L(F_2)}$ such that $Mod_{L(F_1)}(T_1) =_\psi Mod_{L(F_2)}(T_2)$ for some mapping ψ.
For any model I of T_1 we have $A^I \cap B^I = \emptyset$. Hence any model J of T_2 satifies $\psi(A)^J \cap \psi(B)^J = \emptyset$. It is easy to see that this is only possible if either $\psi(A) = \bot$ or $\psi(B) = \bot$ is an axiom in T_2. But then the models I of T_1 with $A^I \neq \emptyset \neq B^I$ do not have corresponding models of T_2. This is a contradiction to $Mod_{L(F_1)}(T_1) =_\psi Mod_{L(F_2)}(T_2)$.

The problem with Definition 4.6 is that the connection is lost between the extensions of different terms with respect to the same interpretation. This shortcoming can be mended by considering n-tupels $\underline{A} = (A_1,...,A_n) \in \mathbf{F}^n$ of terms instead of single terms.

Definition 4.10. Let $\mathbf{F_1}$ and $\mathbf{F_2}$ be two sets of concept and role terms.
(1) We say that $\underline{A} \in \mathbf{F_1}^n$ is *equivalent* to $\underline{B} \in \mathbf{F_2}^n$ ($\underline{A} \equiv \underline{B}$) iff $\{ (A_1^I,...,A_n^I)$; where I is an interpretation $\} = \{ (B_1^J,...,B_n^J)$; where J is an interpretation $\}$.

[21] That means that D only contains primitive concepts and roles, i.e., T is unfolded.
[22] See Proposition 4.11.

(2) $\mathbf{F_1}$ *can be expressed by* $\mathbf{F_2}$ iff for all $n \geq 1$ and all $\underline{A} \in \mathbf{F_1}^n$ there exists $\underline{B} \in \mathbf{F_2}^n$ such that $\underline{A} \equiv \underline{B}$.

(3) $\mathbf{F_1}$ and $\mathbf{F_2}$ have *the same expressive power* iff $\mathbf{F_1}$ can be expressed by $\mathbf{F_2}$ and vice versa.

Proposition 4.11. $\mathbf{F_1}$ can be expressed by $\mathbf{F_2}$ according to Definition 4.10 if and only if $\mathbf{L(F_1)}$ can be expressed by $\mathbf{L(F_2)}$ according to Definition 3.2.

Proof. (1) Assume that $\mathbf{L(F_1)}$ can be expressed by $\mathbf{L(F_2)}$ according to Definition 3.2. Let $\underline{A} = (A_1,\ldots,A_n) \in \mathbf{F_1}^n$ be an n-tuple of concept and role terms. We have to find an n-tuple $\underline{B} \in \mathbf{F_2}^n$ such that $\underline{A} \equiv \underline{B}$.

Let $C_1, \ldots, C_n$ be concept or role names not occurring in $\underline{A}$, where C_i is a concept name iff A_i is a concept term. The T-box $T := \{ C_1 = A_1, \ldots, C_n = A_n \}$ is an element of $L(F_1)$. Thus there exists a T-box T' of $\mathbf{L(F_2)}$ and a mapping ψ: Pred(T) $\rightarrow$ Pred(T') such that T is expressed by T' w.r.t. ψ.

For all i, $1 \leq i \leq n$, the concept or role term B_i is defined as follows. If $\psi(C_i)$ is a primitive concept or role in T', then $B_i := \psi(C_i)$. If $\psi(C_i)$ is a defined concept or role and $\psi(C_i) = D_i$ is its defining axiom in T', then $B_i := D_i$. We have $\underline{B} \in \mathbf{F_2}^n$ since the B_i are atomic terms or terms occurring as right hand side of an axiom in T'. It remains to be shown that $\underline{A} \equiv \underline{B}$.

(1.1) Let I be an interpretation. Since we are only interested in the extensions $A_1^I, \ldots, A_n^I$, and since the concept and role names $C_1, \ldots, C_n$ do not occur in $\underline{A}$, we may without loss of generality assume that $C_1^I = A_1^I, \ldots, C_n^I = A_n^I$. That means that I is a model of T. Since T is expressed by T' w.r.t. ψ, there exists a model J of T' such that $R^I = \psi(R)^J$ for all $R \in$ Pred(T). We have $A_i^I = C_i^I = \psi(C_i)^J = B_i^J$. In fact, if $\psi(C_i)$ is a primitive concept or role in T', then $\psi(C_i)^J = B_i^J$ since $B_i = \psi(C_i)$. If $\psi(C_i)$ is a defined concept or role, then $\psi(C_i)^J = B_i^J$ since J is a model of T' and $\psi(C_i) = B_i$ is an axiom of T'.

Thus we have shown that $\{ (A_1^I,\ldots,A_n^I)$; where I is an interpretation $\} \subseteq \{ (B_1^J,\ldots,B_n^J)$; where J is an interpretation $\}$.

(1.2) Let J be an interpretation. The definition of the languages $\mathbf{L(F)}$ implies that the terms $B_1, \ldots, B_n$ do not contain any defined concepts or roles of T'. Thus, if we are only interested in $(B_1^J,\ldots,B_n^J)$, we may without loss of generality assume that J is a model of T'. We can now proceed as in (1.1).

(2) Assume that $\mathbf{F_1}$ can be expressed by $\mathbf{F_2}$ according to Definition 4.10. Let $T = \{ C_1 = D_1, \ldots, C_n = D_n \}$ be a T-box of $\mathbf{L(F_1)}$ containing the defined concepts and roles $C_1, \ldots, C_n$, and the primitive concepts and roles $P_1, \ldots, P_m$. Please note that, by the definition of $\mathbf{L(F_1)}$, the terms $D_1, \ldots, D_n$ may only contain primitive concepts and roles. We consider the tuple $(D_1,\ldots,D_n,P_1,\ldots,P_m)$, which is an element of $\mathbf{F_1}^{n+m}$. By the assumption, there exists a tuple $(E_1,\ldots,E_n,Q_1,\ldots,Q_m)$ in $\mathbf{F_1}^{n+m}$ such that $(D_1,\ldots,D_n,P_1,\ldots,P_m) \equiv (E_1,\ldots,E_n,Q_1,\ldots,Q_m)$.

Let $A_1, \ldots, A_n, B_1, \ldots, B_m$ be concept and role names not occurring in $(E_1,\ldots,E_n,Q_1,\ldots,Q_m)$, where A_i (B_i) is a concept name iff E_i (Q_i) is a concept term. The T-box T' is defined as $\{ A_1 = E_1, \ldots, B_m = Q_m \}$. We define the mapping ψ: Pred(T) $\rightarrow$ Pred(T') by $\psi(C_i) := A_i$ and $\psi(P_j) := B_j$. It remains to be shown that T is expressed by T' w.r.t. ψ.

(2.1) Assume that I is a model of T. Since $\mathbf{F_1}$ can be expressed by $\mathbf{F_2}$, there exists an interpretation J such that (*) $(D_1^I,\ldots,D_n^I,P_1^I,\ldots,P_m^I) = (E_1^J,\ldots,E_n^J,Q_1^J,\ldots,Q_m^J)$. Since $A_1, \ldots, A_n, B_1, \ldots, B_m$ do not occur in $(E_1,\ldots,E_n,Q_1,\ldots,Q_m)$, we may – without losing property (*) – assume that J is a model of T'. We have $C_i^I = D_i^I$ since I is a model of T, $D_i^I = E_i^J$ because of property (*), and $E_i^J = A_i^J$ since J is a model of T'. This yields $C_i^I = \psi(C_i)^J$; and $P_i^I = \psi(P_i)^J$ can be shown similarly.

(2.2) Assume that J is a model of T'. Since $\mathbf{F_1}$ can be expressed by $\mathbf{F_2}$, there exists an interpretation I such that (∗) $(D_1^I,\ldots,D_n^I,P_1^I,\ldots,P_m^I) = (E_1^J,\ldots,E_n^J,Q_1^J,\ldots,Q_m^J)$. Since C_1, …, C_n do not occurr in $(D_1,\ldots,D_n,P_1,\ldots,P_m)$, we may – without losing property (∗) – assume that I is a model of T. We can now proceed as in (2.1).

5. Examples

This section contains some more examples to demonstrate how Definition 3.2 can be used to prove that one KR1-languages is more expressive than another. In the first example, we shall see how Schild's argument in favour of the expressive power of $\mathcal{U}$ fits into the framework of Definition 3.2. The second example shows that the exists-restrictions in $\mathcal{FL}^-$ are responsible for an increase of expressive power when we go from $\mathcal{FL}_0$ to $\mathcal{FL}^-$. We have already seen in Lemma 2.6, that these additional exists-restrictions do not increase the complexity of subsumption determination. In the third example, we shall see that augmenting $\mathcal{FL}^-$ by exists-in restrictions increases the expressive power. This yields a formal proof for the fact that $\mathcal{FL}$ is more expressive than $\mathcal{FL}^-$.[23] The forth example is concerned with cyclic terminologies.

Example 5.1. In Schild (1988) there is an example of a T-box T_1 of the language $\mathcal{U}$ which contains a concept A (Schild calls it "mankind without rebirth") such that A^I is infinite for all models I of T_1. The T-box T_1 is consistent, i.e., it has at least one model. Let **L** be a KR1-language such that, for all $\Gamma \in$ L, the class $Mod_L(\Gamma)$ of admissible models of Γ is either empty or contains at least one model with finite domain.[24] Then $\mathcal{U}$ cannot be expressed by **L**.
Assume that $\Gamma \in$ L expresses T_1 w.r.t. ψ: $Pred(T_1) \to Pred(\Gamma)$. $Mod_L(\Gamma)$ cannot be empty since T_1 is consistent. Hence there is $J \in Mod_L(\Gamma)$ with finite domain dom(J). Let I be the corresponding model of T_1 with $I \subset_\psi J$. But $\psi(A)^J \subseteq dom(J)$ is finite and A^I is infinite, which is a contradiction since $I \subset_\psi J$ implies $A^I = \psi(A)^J$.

Example 5.2. In Lemma 2.6 we have seen that subsumption determination in $\mathcal{FL}^-$ can be reduced to subsumption determination in $\mathcal{FL}_0$: for any T-box T in $\mathcal{FL}^-$ one can construct a T-box T_0 in $\mathcal{FL}_0$ such that T_0 and T have the same size and for all concept names A, B in T, $A \sqsubseteq_T B$ iff $A \sqsubseteq_{T_0} B$.
Nevertheless, T is not expressed by T_0 in the sense of Definition 3.2. We shall now show that $\mathcal{FL}^-$ cannot be expressed by $\mathcal{FL}_0$. This means that the additional exists-restrictions in $\mathcal{FL}^-$ increase the expressive power of the language. In the proof we shall need the following lemma.

Lemma 5.2.1. Let T be a T-box of $\mathcal{FL}_0$ and let A be a concept name occurring in T. Let I and J be models of T with the same domain, such that I and J coincide on all primitive concepts and for all roles[25] S, $S^J \subseteq S^I$. Then $A^I \subseteq A^J$.
Proof. If A is primitive then $A^I = A^J$. Otherwise, T contains (w.l.o.g.) a terminological axiom of the form $A = \forall S_{11}\forall S_{12}\ldots\forall S_{1n_1}{:}P_1 \sqcap \ldots \sqcap \forall S_{k1}\forall S_{k2}\ldots\forall S_{kn_k}{:}P_k$, where the P_j

[23]The $\mathcal{FL}$-term (SOME (RESTRICT R C)) has the same semantics as our exists-in construct ∃R:C.
[24]Many languages satisfy this property, e.g., $\mathcal{FL}$ and $\mathcal{FL}^-$ of Levesque-Brachman (1987), or $\mathcal{TF}$ and $\mathcal{NTF}$ of Nebel (1989).
[25]In $\mathcal{FL}_0$ all roles are primitive.

are primitive concepts.[26] Assume that an element b_0 of dom(J) is not in A^J. Then there exists j, $1 \le j \le k$, and $b_1, ..., b_{n_j} \in$ dom(J) such that $(b_0,b_1) \in S_{j1}{}^J, ..., (b_{n_j-1},b_{n_j}) \in S_{jn_j}{}^J$ and $b_{n_j} \notin P_j{}^J$. But then $(b_0,b_1) \in S_{j1}{}^I, ..., (b_{n_j-1},b_{n_j}) \in S_{jn_j}{}^I$ and $b_{n_j} \notin P_j{}^I$. This shows $b_0 \notin A^I$.

Consider the T-box $T_1 = \{ A = \exists R \}$, which is an admissible T-box of $\mathcal{FL}^-$. Assume that T_1 is expressed by $T_2 \in \mathcal{FL}_0$ w.r.t. ψ. Let I be the model of T_1 defined by dom(I) := { a, b } and $R^I := \{ (a,b) \}$. Then $A^I = \{ a \}$. Since I is a model of T_1 there exists a model J of T_2 with $I \subset_\psi J$. Let J' be the model of T_2 which coincides with J on all primitive concepts and on all roles except $\psi(R)$, and which interprets $\psi(R)$ as the empty set.
Lemma 5.2.1 implies that $\psi(A)^J \subseteq \psi(A)^{J'}$. Because T_1 is expressed by T_2 w.r.t. ψ, there exists a model I' of T_1 such that $I' \subset_\psi J'$. But $R^{I'} = \psi(R)^{J'} = \emptyset$ and $a \in \psi(A)^J \subseteq \psi(A)^{J'} = A^{I'} = (\exists R)^{I'}$ is a contradiction. This shows that the T-box T_1 of $\mathcal{FL}^-$ cannot be expressed by any T-box of $\mathcal{FL}_0$.

Example 5.3. We shall now show that the T-box $T_1 := \{ A = \exists R: \exists S: P \}$, which uses exists-in-restrictions, cannot be expressed by a T-box of $\mathcal{FL}^-$. Before we can start with the proof, we need a lemma, which is similar to Lemma 5.2.1 above.

Lemma 5.3.1. Let T be a T-box of $\mathcal{FL}^-$ and let A be a concept name occurring in T. Let I and J be models of T with the same domain, such that I and J coincide on all primitive concepts. Assume that for all roles S and all $a \in$ dom(I), $S^J \subseteq S^I$ and $\{ b; (a,b) \in S^I \} \neq \emptyset$ implies $\{ b; (a,b) \in S^J \} \neq \emptyset$. Then $A^I \subseteq A^J$.
Proof. Similar to the proof of Lemma 5.2.1.

Assume that T_1 is expressed by $T_2 \in \mathcal{FL}^-$ w.r.t. ψ. Let I be the model of T_1 defined by dom(I) := { a, b, c, d } := P^I, $S^I := \{ (b,c) \}$ and $R^I := \{ (a,b), (a,d) \}$. Then $A^I = \{ a \}$.
Let J be a model of T_2 with $I \subset_\psi J$. Let J' be the model of T_2 which coincides with J on all primitive concepts and roles with the exception of $\psi(R)$. The extension of $\psi(R)$ w.r.t. J' is defined as $\psi(R)^{J'} := \psi(R)^J \setminus \{ (a,b) \}$. By Lemma 5.3.1, $a \in \psi(A)^J \subseteq \psi(A)^{J'}$.
Let I' be a model of T_1 with $I' \subset_\psi J'$. Then $S^{I'} = \{ (b,c) \}$ and $R^{I'} = \{ (a,d) \}$. But we have also $a \in \psi(A)^{J'} = A^{I'} = (\exists R: \exists S: P)^{I'}$. This is a contradiction, since the element "d" – which is the only $R^{I'}$-successor of the element "a" – does not have an $S^{I'}$-successor.

Example 5.4. Nebel's normal-form language $\mathcal{NTF}$ (see Nebel (1989), p. 59 ff.) is defined as follows: it allows concept and role conjunctions, all-in-restrictions, number-restrictions and negation of undefined concepts. The T-boxes contain only complete definitions.
Let $\mathbf{L_1}$ be the language which contains only the acyclic T-boxes of $\mathcal{NTF}$ and takes all first-order models as admissible models. On the other hand, let $\mathbf{L_2}$ be the language, which contains all T-boxes of $\mathcal{NTF}$ and allows only the greatest fixed-point models (gfp-models) as admissible models of cyclic T-boxes.[27] Then $\mathbf{L_1}$ is a sublanguage of $\mathbf{L_2}$.
(1) The T-box $T := \{ A = (\forall R{:}A) \sqcap (\exists_{\geq 1} S) \}$ is an element of L_2. Considered with gfp-semantics, this T-box says something about the reflexive, transitive closure of R: if R^* denotes

[26]See Proposition 3 in Nebel (1989a).
[27]See Nebel (1989), Chapter 5, and Baader(1990a,b) for the definition and for properties of gfp-models.

the reflexive, transitive closure of R[28], then T means the same as $\{ A = \forall R^*: \exists_{\geq 1} S \}$. This is an immediate consequence of Proposition 7.1 in Baader (1990a). A gfp-model of T is uniquely determined by the extensions of R, S. On the other hand, any partial interpretation – which consists of a domain and extensions for R and S – can be extended to a gfp-model of T.
We shall now prove that the T-box T cannot be expressed by a T-box of $\mathbf{L_1}$. This shows that cycles really add something to the expressive power.
(2) Assume that T' is a T-box of $\mathbf{L_1}$ such that T is expressed by T' w.r.t. ψ: Pred(T) $\rightarrow$ Pred(T'). Since T' is acyclic, we may without loss of generality assume that it is unfolded. That means that the role terms occurring on the right hand sides of axioms of T' are of the form $Q_1 \sqcap \ldots \sqcap Q_n$, where the Q_i are primitive roles. The concept terms on the right hand sides of axioms of T' contain only primitive concepts and roles. It is easy to see that the concept terms $\forall R:(B \sqcap C)$ and $(\forall R:B) \sqcap (\forall R:C)$ are equivalent. Hence any concept term can be transformed into a finite conjunction of terms of the form $\forall R_1:\forall R_2:\ldots\forall R_n:C$, where the R_i are role terms of T', and C has one of the following forms:

(2.1) $C = P$ for a primitive concept P, or
(2.2) $C = \neg P$ for a primitive concept P, or
(2.3) $C = \exists_{\geq n} Q$ for a role term Q, or
(2.4) $C = \exists_{\leq n} Q$ for a role term Q.

We shall abbreviate the prefix "$\forall R_1:\forall R_2:\ldots\forall R_n$" by "$\forall W$" where $W = R_1 R_2 \ldots R_n$ is a word over the set of role terms occurring in T'. In the case n = 0 we also write "$\forall \varepsilon$:C"[29] instead of simply "C". For an interpretation J and a word $W = R_1R_2\ldots R_n$, W^J denotes the composition $R_1^J \circ R_2^J \circ \ldots \circ R_n^J$ of the binary relations $R_1^J, R_2^J, \ldots, R_n^J$. The term ε^J denotes the identity relation, i.e., $\varepsilon^J = \{ (d,d); d \in dom(J) \}$.
(3) Let I be the gfp-model of T defined by $dom(I) := \{ b_i; i \geq 0 \} \cup \{ c_i; i \geq 0 \}$, $R^I := \{ (b_i,b_{i+1}); i \geq 0 \}$, and $S^I := \{ (b_i,c_i); i \geq 0 \}$. Using Proposition 7.1 of Baader (1990a), it is easy to see that $A^I = \{ b_i; i \geq 0 \}$.
(4) Let J be a model of T' such that $I \subset_\psi J$. Without loss of generality we may assume that $\psi(R)$ and $\psi(S)$ are defined roles in T'. The defining axioms for $\psi(R)$ and $\psi(S)$ are of the form

$$\psi(R) = R_1 \sqcap \ldots \sqcap R_k \sqcap Q_1 \sqcap \ldots \sqcap Q_s,$$
$$\psi(S) = S_1 \sqcap \ldots \sqcap S_r \sqcap Q_1 \sqcap \ldots \sqcap Q_s,$$

where $R_1, \ldots, R_k$ ($S_1, \ldots, S_r$) are primitive roles not occurring in the definition of S (R), and $Q_1, \ldots, Q_k$ are the primitive roles occurring in both definitions.
Since $R^I = \psi(R)^J$, $S^I = \psi(S)^J$, and R^I and S^I are nonempty and disjoint, we know that $k \geq 1$ and $r \geq 1$.
(5) The model J' of T' is defined as follows: dom(J') := dom(J); $P^{J'} := P^J$ for all primitive concepts P; $R_1^{J'} := \ldots := R_k^{J'} := \psi(R)^J$, $S_1^{J'} := \ldots := S_k^{J'} := \psi(S)^J$, $Q_1^{J'} := \ldots := Q_k^{J'} := \psi(R)^J \cup \psi(S)^J$; and $Q^{J'} := \emptyset$ for all the other primitive roles.
Let I' be a model of T such that $I' \subset_\psi J'$. We have $R^{I'} = \psi(R)^{J'} = \psi(R)^J = R^I$, and $S^{I'} = \psi(S)^{J'} = \psi(S)^J = S^I$. Thus $\psi(A)^{J'} = A^{I'} = A^I = \{ b_i; i \geq 0 \}$.

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[28]That means that for all interpretations I, the extension of R^* w.r.t. I is defined as the reflexive, transitive closure of R^I.
[29]"ε" denotes the empty word.

(6) Let t be a nonnegative integer, and let J" be the model of T' which coincides with J' on the extensions of all primitive concepts and roles with the exception of S_1: $S_1^{J''} := S_1^{J'} \setminus \{ (b_t,c_t) \}$. Let I" be a model of T such that $I'' \subset_\psi J''$. We have $R^{I''} = \psi(R)^{J''} = \psi(R)^{J'} = R^I$, and $S^{I''} = \psi(S)^{J''} = \psi(S)^{J'} \setminus \{ (b_t,c_t) \} = S^I \setminus \{ (b_t,c_t) \}$. Thus $\psi(A)^{J''} = A^{I''} = \{ b_i; i > t \}$.
In particular, we have $b_0 \notin \psi(A)^{J''}$. We have seen in (2) that the defining axiom for $\psi(A)$ is of the form $\psi(A) = D$ where D is a conjunction of terms $\forall W{:}C$ as described in (2). Assume that the term $\forall W{:}C$ is responsible for $b_0 \notin \psi(A)^{J''}$. That means that there exists an individual $e \in dom(J'')$ such that $b_0 W^{J''} e$ and $e \notin C^{J''}$. Obviously, we also have $b_0 W^{J'} e$; and since $b_0 \in \psi(A)^{J''}$ this yields $e \in C^{J'}$.
The fact that $e \notin C^{J''}$ and $e \in C^{J'}$ implies that C is of the form $\exists_{\geq n} Q$ for a role term Q. In fact, C cannot be of the form (2.1) or (2.2) since the J"-extensions of all primitive concepts are the same as their J'-extensions. The term C cannot be of the form (2.4) since the J"-extensions of all role terms are contained in their J'-extensions.
(7) The only individual having less role successors in J" than in J' is the individual b_t. Thus $e \notin (\exists_{\geq n} Q)^{J''}$ and $e \in (\exists_{\geq n} Q)^{J'}$ implies that $e = b_t$. The definition of the role extensions in J' and J" implies that the J"-extension of any role term in T' is contained in the relation $R^I \cup S^I$. Thus $b_0 W^{J''} b_t$ yields $|W| = t$. This is a contradiction if we choose t big enough.

In Baader (1990a), I have also considered cyclic terminologies for a small language (only concept conjunction and all-in-restrictions are allowed) w.r.t. three different kinds of semantics: greatest fixed-point semantics, least fixed-point semantics, and what Nebel (1989) calls descriptive semantics. It turns out that these three kinds of semantics are incomparable w.r.t. expressive power, i.e., it is not the case that one of them can be expressed by another one. All three are more expressive than the corresponding language which allows only acyclic T-boxes (that is the language $\mathcal{TL}$, which was defined in Proposition 2.5).

6. Conclusion

This paper provides a definition of expressive power of KR-languages that depends only on the model-theoretic semantics for the languages. The definition enables us to compare different KR-languages without any reference to purely intuitive arguments. This does not mean that the results which can be obtained by using this definition are counterintuitive. On the contrary, the examples show that they coincide with what has informally been claimed. But now intuition can be backed up by formal proofs. We have concentrated more on negative proofs – which demonstrate that a given language cannot be expressed by another language – because for those a formal definition of expressive power is indispensable. It turned out that finding negative proofs is rather hard, even for examples where a difference in expressive power seems to be intuitively clear. This indicates that comparison of expressive power is a complex problem where one should not solely rely on intuition.

Though we have only considered KL-ONE-based KR-language, our notion of expressive power can be used for all KR-languages with model-theoretic semantics. It provides an important tool for the investigation of differences and similarities between various KR-languages. In the design of KR-systems it can be used to avoid redundant constructs and to decide what kind of constructs should be included.

The definition of expressiveness also sheds a new light on the tradeoff between expressive power and tractability of reasoning. We have seen that there are KR-languages which have the same expressive power but different complexity for subsumption determination. This shows that it is not enough to consider complexity results for the languages. The compactness with which the language can express things is also very important. A KR-language may be able to express things in a very compact form – which is convenient for the user – but may have, just for that reason, a bad complexity behaviour for the reasoning component. Nevertheless, this language should be preferred to an equivalent language which needs a large set of formulas to describe the same thing, and has – with respect to this already very large input – a better complexity behaviour. Sometimes, the tradeoff lies between convenience and computational tractability.

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DFKI Research Reports

RR-90-01
Franz Baader
Terminological Cycles in KL-ONE-based Knowledge Representation Languages
33 pages

Abstract: Cyclic definitions are often prohibited in terminological knowledge representation languages, because, from a theoretical point of view, their semantics is not clear and, from a practical point of view, existing inference algorithms may go astray in the presence of cycles. In this paper we consider terminological cycles in a very small KL-ONE-based language. For this language, the effect of the three types of semantics introduced by Nebel (1987, 1989, 1989a) can be completely described with the help of finite automata. These descriptions provide a rather intuitive understanding of terminologies with cyclic definitions and give insight into the essential features of the respective semantics. In addition, one obtains algorithms and complexity results for subsumption determination. The results of this paper may help to decide what kind of semantics is most appropriate for cyclic definitions, not only for this small language, but also for extended languages. As it stands, the greatest fixed-point semantics comes off best. The characterization of this semantics is easy and has an obvious intuitive interpretation. Furthermore, important constructs – such as value-restriction with respect to the transitive or reflexive-transitive closure of a role – can easily be expressed.

RR-90-02
Hans-Jürgen Bürckert
A Resolution Principle for Clauses with Constraints
25 pages

Abstract: We introduce a general scheme for handling clauses whose variables are constrained by an underlying constraint theory. In general, constraints can be seen as quantifier restrictions as they filter out the values that can be assigned to the variables of a clause (or an arbitrary formulae with restricted universal or existential quantifier) in any of the models of the constraint theory. We present a resolution principle for clauses with constraints, where unification is replaced by testing constraints for satisfiability over the constraint theory. We show that this constrained resolution is sound and complete in that a set of clauses with constraints is unsatisfiable over the constraint theory iff we can deduce a constrained empty clause for each model of the constraint theory, such that the empty clauses constraint is satisfiable in that model. We show also that we cannot require a better result in general, but we discuss certain tractable cases, where we need at most finitely many such empty clauses or even better only one of them as it is known in classical resolution, sorted resolution or resolution with theory unification.

RR-90-03
Andreas Dengel & Nelson M. Mattos
Integration of Document Representation, Processing and Management
18 pages

Abstract: This paper describes a way for document representation and proposes an approach towards an integrated document processing and management system. The approach has the intention to capture essentially freely structured documents, like those typically used in the office domain. The document analysis system ANASTASIL is capable to reveal the structure of complex paper documents, as well as logical objects within it, like receiver, footnote, date. Moreover, it facilitates the handling of the containing information. Analyzed documents are stored by the management system KRISYS that is connected to several different subsequent services. The described integrated system can be considered as an ideal extension of the human clerk, making his tasks in information processing easier. The symbolic representation of the analysis results allow an easy transformation in a given international standard, e.g., ODA/ODIF or SGML, and to interchange it via global network.

RR-90-04
Bernhard Hollunder & Werner Nutt
Subsumption Algorithms for Concept Languages
34 pages

Abstract: We investigate the subsumption problem in logic-based knowledge representation languages of the KL-ONE family and give decision procedures. All our languages contain as a kernel the logical connectives conjunction, disjunction, and negation for concepts, as well as role quantification. The algorithms are rule-based and can be understood as variants of tableaux calculus with a special control strategy. In the first part of the paper, we add number restrictions and conjunction of roles to the kernel language. We show that subsumption in this language is decidable, and we investigate sublanguages for which the problem of deciding subsumption is PSPACE-complete. In the second part, we amalgamate the kernel language with feature descriptions as used in computational linguistics. We show that feature descriptions do not increase the complexity of the subsumption problem.

RR-90-05
Franz Baader
A Formal Definition for the Expressive Power of Knowledge Representation Languages
22 pages

Abstract: The notions "expressive power" or "expressiveness" of knowledge representation languages (KR-languages) can be found in most papers on knowledge representation; but these terms are usually just used in an intuitive sense. The papers contain only informal descriptions of what is meant by expressiveness. There are several reasons which speak in favour of a formal definition of expressiveness: For example, if we want to show that certain expressions in one language *cannot* be expressed in another language, we need a strict formalism which can be used in mathematical proofs. Though we shall only consider KL-ONE-based KR-language in our motivation and in the examples, the definition of expressive power which will be given in this paper can be used for all KR-languages with model-theoretic semantics. This definition will shed a new light on the tradeoff between expressiveness of a representation language and its computational tractability. There are KR-languages with identical expressive power, but different complexity results for reasoning. Sometimes, the tradeoff lies between convenience and computational tractability. The paper contains several examples which demonstrate how the definition of expressive power can be used in positive proofs – that is, proofs where it is shown that one language can be expressed by another language – as well as for negative proofs – which show that a given language cannot be expressed by the other language.

DFKI Technical Memos

TM-89-01
Susan Holbach-Weber
Connectionist Models and Figurative Speech
27 pages

Abstract: This paper contains an introduction to connectionist models. Then we focus on the question of how novel figurative usages of descriptive adjectives may be interpreted in a structured connectionist model of conceptual combination. The suggestion is that inferences drawn from an adjective's use in familiar contexts form the basis for all possible interpretations of the adjective in a novel context. The more plausible of the possibilities, it is speculated, are reinforced by some form of one-shot learning, rendering the interpretative process obsolete after only one (memorable) encounter with a novel figure of speech.

TM-90-01
Som Bandyopadhyay
Towards an Understanding of Coherence in Multimodal Discourse
18 pages

Abstract: An understanding of coherence is attempted in a multimodal framework where the presentation of information is composed of both text and picture segments (or, audio-visuals in general). Coherence is characterised at three levels: coherence at the syntactic level which concerns the linking mechanism of the adjacent discourse segments at the surface level in order to make the presentation valid; coherence at the semantic level which concerns the linking of discourse segments through some semantic ties in order to generate a wellformed thematic organisation; and, coherence at the pragmatic level which concerns effective presentation through the linking of the discourse with the addressees´ preexisting conceptual framework by making it compatible with the addressees´ interpretive ability, and linking the discourse with the purpose and situation by selecting a proper discourse typology. A set of generalised coherence relations are defined and explained in the context of picture-sequence and multimodal presentation of information.

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Franz Baader

RR-90-05
Research Report