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An Alternative Proof Method for Possibilistic Logic and its Application to Terminological Logics

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Abstract

Possibilistic logic, an extension of first-order logic, deals with uncertainty that can be estimated in terms of possibility and necessity measures. Syntactically, this means that a first-order formula is equipped with a possibility degree or a necessity degree that expresses to what extent the formula is possibly or necessarily true. Possibilistic resolution, an extension of the well-known resolution principle, yields a calculus for possibilistic logic which respects the semantics developed for possibilistic logic.

A drawback, which possibilistic resolution inherits from classical resolution, is that it may not terminate if applied to formulas belonging to decidable fragments of first-order logic. Therefore we propose an alternative proof method for possibilistic logic. The main feature of this method is that it completely abstracts from a concrete calculus but uses as basic operation a test for classical entailment. If this test is decidable for some fragment of first-order logic then possibilistic reasoning is also decidable for this fragment.

We then instantiate possibilistic logic with a terminological logic, which is a decidable subclass of first-order logic but nevertheless much more expressive than propositional logic. This yields an extension of terminological logics towards the representation of uncertain knowledge which is satisfactory from a semantic as well as algorithmic point of view.

Contents

1 **Introduction** 3

2 **Possibilistic Logic** 5

3 **An Alternative Proof Method for Possibilistic Logic** 11

4 **A Possibilistic Extension of Terminological Logics** 18

4.1 Terminological knowledge representation 19

4.2 The possibilistic extension 21

5 **Conclusion** 22

1 Introduction

A wide range of approaches have been proposed for the treatment of uncertainty in Artificial Intelligence applications such as expert systems or knowledge representation systems (for an overview see, e.g., [12, 9]). To deal with uncertainty that can be estimated in terms of possibility and necessity measures (as used in the framework of possibility theory [17]) possibilistic logic is a promising candidate. In fact, a basic feature of possibilistic logic is its ability to model states of knowledge ranging from complete information to total ignorance by expressing lower bounds for the possibility or necessity of some piece of knowledge. This allows one, for instance, to distinguish between the total lack of certainty in the truth of a proposition and the certainty that the proposition is false.

From a syntactical point of view, possibilistic logic employs closed first-order formulas which are equipped with a possibility degree or a necessity degree: A weight $\Pi\,\alpha$ (resp. $\mathrm{N}\,\alpha$) attached to a formula p models to what extent p is possibly (resp. necessarily) true, where α ranges between 0 and 1. To express, for example, that p is likely to be true one may use the necessity-valued formula $(p,\ \mathrm{N}\,0.7)$, whereas one may write $(p,\ \Pi 0.9)$ to model that p is rather possible but not certain at all.

Recently, a semantics for possibilistic logic has been presented for the general case where possibility- as well as necessity-valued formulas are allowed (cf. [10]). The semantics is based on fuzzy sets of interpretations, i.e., to each classical interpretation ω of the first-order formulas occurring in a set of possibilistic formulas a value $\pi(\omega)$ between 0 and 1 is associated. This value indicates the quality of an interpretation: $\pi(\omega) \leq \pi(\omega')$ means that interpretation ω' is at least as acceptable as ω to be the real world. The possibility and necessity of a formula p is then given by

$$\Pi(p) = \sup\{\pi(\omega) \mid \omega \models p\} \quad \text{and} \quad \mathrm{N}(p) = 1 - \Pi(\neg p).$$

A fuzzy set of interpretations satisfies a possibilistic formula $(p,\ \Pi\,\alpha)$ (resp. $(p,\ \mathrm{N}\alpha)$) iff $\Pi(p) \geq \alpha$ (resp. $\mathrm{N}(p) \geq \alpha$). Entailment is then straightforwardly defined as follows: A possibilistic formula ϕ is a logical consequence of a possibilistic knowledge base Φ, i.e. a set of possibilistic formulas, iff every fuzzy set of interpretations satisfying each element in Φ also satisfies ϕ.

Possibilistic resolution (see, e.g., [10])—an extension of the well-known resolution principle to possibilistic logic—yields a calculus for possibilistic reasoning, i.e., to answer the question whether or not a possibilistic formula is entailed by a possibilistic knowledge base. In fact, if applications of the possibilistic resolution rule to $\Phi \cup \{(\neg p,\ \mathrm{N}1)\}$ yield a derivation of an empty possibilistic clause $(\Box, v)$ then Φ entails (p, v), where v is either $\Pi\,\alpha$ or $\mathrm{N}\,\alpha$ for some $\alpha \in [0, 1]$.

A drawback, which possibilistic resolution inherits from classical resolution, is that it may not terminate if applied to formulas belonging to decidable fragments of first-order logic. In fact, if the input formulas contain existential quantifiers in the scope of universal quantifiers, one in general gets (Skolem) function symbols when transforming these formulas into clause form. But this means that resolution (and thus possibilistic resolution) may not terminate.[1] Moreover, the transformation of possibilistic formulas into clause form yields another problem: A set of possibility-valued formulas cannot always be transformed into an "equivalent" set of clauses—even not for the propositional case (cf. [10], Section 3.1).

For these reasons we propose an alternative proof method for possibilistic logic. The main feature of this method is that it completely abstracts from a concrete calculus (such as resolution or tableaux methods), but uses as basic operation a test for classical entailment. If this test is decidable for some fragment of first-order logic, it turns out that possibilistic reasoning is also decidable for this fragment. Additionally, if one has an algorithm solving the entailment problem, our proof method automatically yields an algorithm realizing possibilistic entailment. We prove that the proposed method is sound and complete (for the general case where both possibility- and necessity-valued formulas are allowed) with respect to the semantics for possibilistic logic.

We then show how our method can be utilized to obtain decision procedures for a possibilistic extension of terminological knowledge representation formalisms, also called terminological logics. These formalisms, which are employed in terminological representation systems such as BACK [13], CLASSIC [4], KRIS [2], or LOOM [11] are in general decidable fragments of first-order logic, but are nevertheless expressive enough to define relevant concepts of a problem domain. This is done by building complex concepts from primitive concepts (unary predicates) and roles (binary predicates) with the help of operations provided by the concept language of the particular formalism. For example, if we assume that *person* and *car* are concepts and that *owns* is a role, the concept $\mathit{person} \sqcap \exists \mathit{owns}.\mathit{car}$ describes the set of all persons having some car. Additionally, objects (or individuals) can be introduced by stating that an object is instance of a concept (e.g., $\mathit{Tom}: \mathit{person}$), or that two objects are related by a role (e.g., $(\mathit{Tom}, \mathit{car\_7}) : \mathit{owns}$).

Several approaches have already been proposed to enhance the expressivity of terminological formalisms with (some form of) uncertainty (e.g., probabilistic implications between concepts [7] or subsumption between fuzzy

[1] It should be noted, however, that the resolution calculus can be modified such that it yields decision procedures for various decidable fragments of first-order logic (see e.g. [14]).

concepts [15]). The approach, which comes nearest to ours, is described in [16] and outlines an architecture for incorporating approximate reasoning into terminological systems. The main problem of this approach, however, is that its behavior is only described by means of examples, which means that neither a complete semantics nor algorithms for the main inference problems are given.

An extension of terminological formalisms towards the representation of uncertain knowledge which is satisfactory from a semantic as well as algorithmic point of view can be obtained by instantiating possibilistic logic with a terminological logic. This means that we do not allow arbitrary first-order formulas in possibilistic formulas but only those which can be formed by a particular terminological formalism. To be more precise, in the possibilistic extension we present one can, on the one hand, state plausible rules between concepts. For example, the rule

$$(\mathit{person} \sqcap \mathit{rich} \rightarrow \exists \mathit{owns}.\mathit{Porsche},\ \Pi 0.7),$$

expresses that "rich persons are likely to own a Porsche." Of course, universally valid rules, i.e., strict implications between concepts such as "every Porsche is car" can be formulated by using the maximal necessity value N1. On the other hand, one can express uncertain knowledge concerning particular objects by adding possibility or necessity values to formulas expressing concept and role instanceships.

This approach has not only the advantage of being semantically sound. It also provides one with decision procedures for the basic inference problems (e.g., possibilistic entailment) which are sound and complete with respect to the semantics for possibilistic logic. These decision procedures can immediately be obtained by instantiating our proof method with inference algorithms for terminological logics as, for example, described in [5, 1].

The paper is organized as follows. In Section 2 we introduce syntax and semantics of possibilistic logic. The alternative proof method and the proof of its soundness and completeness are given in Section 3. Finally, in Section 4, we propose a possibilistic extension of terminological logics.

2 Possibilistic Logic

This section reviews possibilistic logic. We start with introducing the syntax for possibilistic formulas, and then we recapitulate the semantics for possibilistic logic as defined in [10]. Finally, possibilistic resolution, a calculus for possibilistic logic, is shortly described.

A *possibilistic formula* is either a pair $(p, \Pi\alpha)$ or $(p, \mathrm{N}\alpha)$ where p is a closed first-order formula and $\alpha \in [0, 1]$. Intuitively, a possibility-valued formula

$(p,\ \Pi\alpha)$ (resp. necessity-valued formula $(p,\ \mathrm{N}\alpha)$) expresses that p is possibly (resp. necessarily) true at least to degree α. A finite set of possibilistic formulas is called a *possibilistic knowledge base.*

As already mentioned in the introduction, the meaning of possibilistic formulas is defined in terms of fuzzy sets of interpretations. This means that to each (classical) interpretation ω of the first-order formulas occurring in a possibilistic knowledge base a value $\pi(\omega)$ between 0 and 1 is associated. This value indicates the quality of an interpretation, i.e., $\pi(\omega) < \pi(\omega')$ means that interpretation ω' prevails over ω to be the real world.

To be more formal, let Φ be a possibilistic knowledge base and let Ω be the set of interpretations of $\widehat{\Phi} = \{p \mid (p,\ \mathrm{N}\,\alpha) \in \Phi \text{ or } (p,\ \Pi\,\alpha) \in \Phi\}$. A *possibility distribution* π on Ω is a mapping from Ω to $[0,1]$ such that $\pi(\omega) = 1$ for some $\omega \in \Omega$. This normalization requirement guarantees that there is at least one world which could be considered as the real one. Every possibility distribution π on Ω induces two functions, denoted by Π' and N', mapping elements of $\widehat{\Phi}$ to $[0,1]$. These functions, called *possibility measure* and *necessity measure*, are defined as follows. If p is a formula in $\widehat{\Phi}$ then

- $\Pi'(p) = \sup\{\pi(\omega) \mid \omega \in \Omega \text{ and } \omega \models p\}$ and
- $\mathrm{N}'(p) = \inf\{1 - \pi(\omega) \mid \omega \in \Omega \text{ and } \omega \not\models p\}$,

where $\sup\{\} := 0$ and $\inf\{\} := 1$.

Thus, if $\Pi'(p) = \alpha$ there is an interpretation ω such that $\omega \models p$ and $\pi(\omega) = \alpha$, or there is an infinite sequence $\omega_0, \omega_1, \omega_2, \ldots$ of interpretations such that $\omega_i \models p$, $\pi(\omega_i) < \pi(\omega_{i+1})$, and α is the least upper bound of $\{\pi(\omega_0), \pi(\omega_1), \pi(\omega_2), \ldots\}$. Conversely, if $\mathrm{N}'(p) = \alpha$ then for all ω such that $\omega \not\models p$ we have $\pi(\omega) \leq 1 - \alpha$.

An immediate consequence of this definition is

$$\Pi'(p \vee q) = \max\{\Pi'(p),\ \Pi'(q)\} \quad \text{and} \quad \Pi'(p \wedge q) \leq \min\{\Pi'(p),\ \Pi'(q)\}, \quad (1)$$

which in fact shows that the possibility measure is in accordance with the basic axioms of possibility theory (cf. [17]). Moreover, by duality of the measures Π' and N', i.e., $\Pi'(p) = 1 - \mathrm{N}'(\neg p)$, we have for the necessity measure

$$\mathrm{N}'(p \wedge q) = \min\{\mathrm{N}'(p),\ \mathrm{N}'(q)\} \quad \text{and} \quad \mathrm{N}'(p \vee q) \geq \max\{\mathrm{N}'(p),\ \mathrm{N}'(q)\}. \quad (2)$$

If a (first-order) formula $p \rightarrow q$ is valid, i.e. $\{p\} \models q$, it is easy to verify that

$$\Pi'(p) \leq \Pi'(q) \quad \text{and} \quad \mathrm{N}'(p) \leq \mathrm{N}'(q); \quad (3)$$

furthermore $\Pi'(\top) = \mathrm{N}'(\top) = 1$ for any tautology $\top$, and $\Pi'(\bot) = \mathrm{N}'(\bot) = 0$ for any contraction $\bot$.

In possibilistic logic, the notions of satisfaction and entailment are defined with respect to possibility distributions. We say that a possibility distribution π on a set Ω of interpretations *satisfies* a possibilistic formula $(p,\ \Pi\alpha)$, written as $\pi \models (p,\ \Pi\alpha)$, iff $\Pi'(p) \geq \alpha$, and it *satisfies* $(p,\ \mathrm{N}\alpha)$, written as $\pi \models (p,\ \mathrm{N}\alpha)$, iff $\mathrm{N}'(p) \geq \alpha$. A possibility distribution π on Ω *satisfies* a possibilistic knowledge base Φ iff $\pi \models \phi$ for all $\phi \in \Phi$. Finally, we say that a possibilistic formula ϕ is *entailed* by a possibilistic knowledge base Φ, denoted by $\Phi \models \phi$, iff $\pi \models \phi$ for all π such that $\pi \models \Phi$ holds.

Example 2.1 Assume that Φ is given by $\{(p,\ \mathrm{N}0.8),\ (p \rightarrow q,\ \mathrm{N}0.4),\ (q \rightarrow r,\ \Pi 0.7)\}$. Then Φ entails the formula $(r,\ \Pi 0.7)$.

To see this, let π be a possibility distribution satisfying Φ. We first observe that $\pi \models (q,\ \mathrm{N}0.4)$. In fact, since $\{p, p \rightarrow q\} \models q$ we conclude

$$\mathrm{N}'(q) \geq \mathrm{N}'(p \wedge (p \rightarrow q)) = \min\{\mathrm{N}'(p),\ \mathrm{N}'(p \rightarrow q)\} \geq 0.4.$$

Duality gives us $\Pi'(\neg q) \leq 0.6$, which means that $\pi(\omega) \leq 0.6$ for all ω with $\omega \models \neg q$. Since $\pi \models (q \rightarrow r,\ \Pi 0.7)$, we have

$$0.7 \leq \sup\{\pi(\omega) \mid \omega \models q \rightarrow r\} = \sup\{\pi(\omega) \mid \omega \models r\}$$

because $\pi(\omega) \leq 0.6$ for all ω with $\omega \models \neg q$. This shows that π satisfies $(r,\ \Pi 0.7)$, and we can conclude that $(r,\ \Pi 0.7)$ is entailed by Φ.

There are possibilistic knowledge bases which are not satisfied by any possibilistic distribution. For example, if Φ contains both $(p,\ \mathrm{N}0.7)$ and $(\neg p,\ \mathrm{N}0.4)$, one gets $\Pi'(\neg p) = \sup\{\pi(\omega) \mid \omega \models \neg p\} \leq 0.3$ and $\Pi'(p) = \sup\{\pi(\omega) \mid \omega \models p\} \leq 0.6$ for every possibility distribution π. But this means that $\pi(\omega) \leq 0.6$ for every ω, which shows that the normalization requirement, i.e., $\pi(\omega) = 1$ for some ω, is not satisfied. Thus Φ cannot be satisfied by any possibilistic distribution. This, of course, means that *every* possibilistic formula is entailed by Φ. However, the fact that we have more confidence in the truth of p than in the truth of $\neg p$ is not sanctioned by the semantics just described.

In order to achieve a better behavior, i.e., to avoid that any possibilistic formula is entailed by an "inconsistent" possibilistic knowledge base, the semantics given above is slightly extended.

Let Ω be a set of interpretations and let $\omega_\perp$ be an *absurd interpretation* such that $\omega_\perp \models p$ for all formulas p. A possibility distribution is now a mapping from $\Omega_\perp := \Omega \cup \{\omega_\perp\}$ to $[0,1]$ such that $\pi(\omega) = 1$ for some $\omega \in \Omega_\perp$. The possibility measure Π and necessity measure N induced by a possibility distribution π on $\Omega_\perp$ is defined by

- $\Pi(p) = \sup\{\pi(\omega) \mid \omega \in \Omega_\perp$ and $\omega \models p\}$ and

- $N(p) = \inf\{1 - \pi(\omega) \mid \omega \in \Omega_\bot \text{ and } \omega \not\models p\}$,

where p is a first-order formula.

Observe that $\Pi(p) = \max\{\Pi'(p), \pi(\omega_\bot)\}$ and $N(p) = N'(p)$, which means that the duality between Π and N can be expressed by

$$\Pi(p) = \max\{1 - N(\neg p), \pi(\omega_\bot)\}.$$

Furthermore, it can easily be verified that the thus defined possibility and necessity measures satisfy the basic axioms (1), (2), and (3) previously introduced.

Satisfaction and entailment are defined as before except that we now consider possibility distributions on $\Omega_\bot$ (instead of Ω).

In [10] it has already been mentioned that both semantics coincide for possibilistic knowledge bases that are "consistent." To be more precise, suppose that there is a possibilistic distribution π on Ω satisfying Φ. Then ϕ is entailed by Φ according to the first semantics if and only if ϕ is entailed by Φ according to the modified, inconsistency tolerant, semantics. Of course, both semantics differ in case Φ is inconsistent. On the one hand, recall that Φ given by $\{(p, \text{N}0.7), (\neg p, \text{N}0.4)\}$ entails every possibilistic formula according to the first semantics. On the other hand, according to the inconsistency tolerant semantics we have $\Phi \models (p, \text{N}0.7)$ and $\Phi \models (\neg p, \text{N}0.4)$, but, which can easily be checked, $\Phi \not\models (p, \text{N}\alpha)$ for $\alpha > 0.7$ and $\Phi \not\models (\neg p, \text{N}\alpha')$ for $\alpha' > 0.4$. This shows that one cannot longer derive any possibilistic formula from an inconsistent possibilistic knowledge base.

A possibilistic knowledge base that is inconsistent according to the first semantics is *more or less inconsistent* according to the inconsistency tolerant semantics. For example, $\{(p, \text{N}\alpha), (\neg p, \text{N}\alpha)\}$ should be considered more inconsistent than $\{(p, \text{N}\beta), (\neg p, \text{N}\beta)\}$ if $\alpha > \beta$. To measure the strength of inconsistency the following definition has been introduced in [10].

Definition 2.2 *The* inconsistency degree *of a possibilistic knowledge base Φ, Incons(Φ), is defined as follows:*

- *If there is a possibility distribution π on $\Omega_\bot$ such that $\pi \models \Phi$ and $\pi(\omega) = 1$ for some $\omega \in \Omega$, then Φ is* possibly inconsistent *and Incons(Φ) = $\Pi\alpha$ where $\alpha = \inf\{\pi(\omega_\bot) \mid \pi \models \Phi\}$. If Incons($\Phi$) = $\Pi 0$ we say that Φ is* completely consistent.

- *If for all possibility distributions π on $\Omega_\bot$, $\pi \models \Phi$ implies $\pi(\omega) < 1$ for every $\omega \in \Omega$, Φ is* necessarily inconsistent *and Incons(Φ) = $\text{N}\alpha$ where $\alpha = \inf\{1 - \pi(\omega) \mid \omega \in \Omega \text{ and } \pi \models \Phi\}$.*

To illustrate this definition let us consider some examples. An example for a *completely consistent* knowledge base is given in Example 2.1, i.e., $\Phi = \{(p,\ \mathrm{N}0.8), (p \rightarrow q,\ \mathrm{N}0.4), (q \rightarrow r,\ \Pi 0.7)\}$. In fact, it is easy to verify that the possibilistic distribution π defined by

$$\pi(\omega) = \begin{cases} 0 & \text{if } \omega = \omega_\perp \\ 0.2 & \text{if } \omega \not\models p \\ 0.6 & \text{if } \omega \not\models p \rightarrow q \\ 1 & \text{otherwise} \end{cases}$$

satisfies every formula in Φ.

To determine the inconsistency degree of $\Phi_1 = \{(p,\ \mathrm{N}\alpha), (\neg p,\ \Pi\beta)\}$ we construct an appropriate possibility distribution π on $\Omega_\perp$ satisfying Φ_1. If $\pi \models \Phi_1$ then $\pi(\omega) \leq 1 - \alpha$ for every interpretation ω with $\omega \not\models p$ (because $\mathrm{N}(p) = 1 - \sup\{\pi(\omega) \mid \omega \in \Omega_\perp \text{ and } \omega \not\models p\} \geq \alpha$). First assume that $\alpha + \beta \leq 1$. We observe that the possibility distribution defined by

$$\pi(\omega) = \begin{cases} 1 & \text{if } \omega \not\models \neg p \\ \beta & \text{if } \omega \not\models p \\ 0 & \text{if } \omega = \omega_\perp \end{cases}$$

satisfies Φ_1. In fact, $\mathrm{N}(p) = 1 - \sup\{\pi(\omega) \mid \omega \in \Omega_\perp \text{ and } \omega \not\models p\} = 1 - \beta \geq \alpha$ and $\Pi(\neg p) = \sup\{\pi(\omega) \mid \omega \in \Omega_\perp \text{ and } \omega \models \neg p\} \geq \beta$, which shows that $\pi \models \Phi_1$. Thus Φ_1 is *completely consistent* if $\alpha + \beta \leq 1$. Now assume that $\alpha + \beta > 1$. Recall that $\pi(\omega) \leq 1 - \alpha$ for every ω with $\omega \not\models p$, which shows that $\sup\{\pi(\omega) \mid \omega \in \Omega_\perp \text{ and } \omega \not\models p\} < \beta$ (since $1 - \alpha < \beta$). But this means that $\pi(\omega_\perp) \geq \beta$ for all π satisfying Φ_1 because $\Pi(\neg p) \geq \beta$. Thus Φ_1 is *possibly inconsistent*. Since the possibility distribution defined by

$$\pi(\omega) = \begin{cases} 1 & \text{if } \omega \not\models \neg p \\ 1 - \alpha & \text{if } \omega \not\models p \\ \beta & \text{if } \omega = \omega_\perp, \end{cases}$$

satisfies Φ_1 we have $Incons(\Phi_1) = \Pi\beta$.

An example for a *necessarily inconsistent* possibilistic knowledge base is given by $\Phi_2 = \{(p,\ \mathrm{N}\alpha), (\neg p,\ \mathrm{N}\beta)\}$ where $\alpha > 0$ and $\beta > 0$. It can easily be checked that $Incons(\Phi_2) = \mathrm{N}\min\{\alpha, \beta\}$.

The following proposition, which has been proved in [10], shows that the entailment problem in possibilistic logic can be reduced to the problem of determining the inconsistency degree of a possibilistic knowledge base, and vice versa.

Proposition 2.3 (Lang, Dubois, and Prade) *Let Φ be a possibilistic knowledge base. Then:*

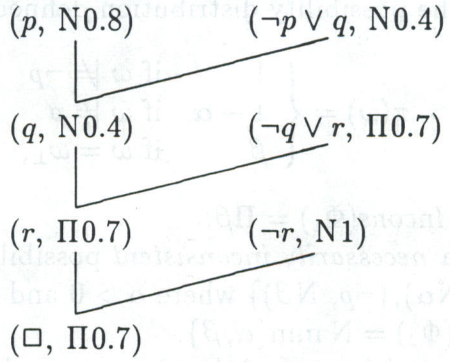
- $\Phi \models (p,\ \Pi\alpha)$ *iff* $\mathit{Incons}(\Phi \cup \{(\neg p,\ \mathrm{N}1)\}) \geq \Pi\alpha$,
- $\Phi \models (p,\ \mathrm{N}\alpha)$ *iff* $\mathit{Incons}(\Phi \cup \{(\neg p,\ \mathrm{N}1)\}) \geq \mathrm{N}\alpha$.

In order to determine (lower bounds for) the inconsistency degree of a possibilistic knowledge base the resolution principle has been extended such that it can be applied to possibilistic formulas (see, e.g., [10]). Let (c, v), (c', v') be possibilistic formulas, where c, c' are first-order formulas in clause form and v, v' are possibility or necessity degrees. The possibilistic resolution rule allows one to derive a possibilistic formula $(\mathit{res}(c, c'), v \circ v')$, where $\mathit{res}(c, c')$ is a classical resolvent of c, c', and $\circ$ is defined by

$$\begin{aligned} \mathrm{N}\alpha \circ \mathrm{N}\alpha' &= \mathrm{N}\min\{\alpha, \alpha'\} \\ \mathrm{N}\alpha \circ \Pi\alpha' &= \begin{cases} \Pi\alpha' & \text{if } \alpha + \alpha' > 1 \\ \Pi 0 & \text{else} \end{cases} \\ \Pi\alpha \circ \Pi\alpha' &= \Pi 0. \end{aligned}$$

If applications of the rule yield a derivation of an empty possibilistic clause $(\Box, w)$ from a set Φ of possibilistic clauses, a lower bound for the inconsistency degree of Φ is given by w, i.e., $\mathit{Incons}(\Phi) \geq w$ (cf. [10]).

Example 2.4 Let us review $\Phi = \{(p,\ \mathrm{N}0.8), (\neg p \vee q,\ \mathrm{N}0.4), (\neg q \vee r,\ \Pi 0.7)\}$ of Example 2.1. In order to show that $\Phi \models (r,\ \Pi 0.7)$ one starts with $\Phi \cup \{(\neg r,\ \mathrm{N}1)\}$. Then possibilistic resolution yields the following derivation of the empty possibilistic clause:



Since $\mathit{Incons}(\Phi \cup \{(\neg r,\ \mathrm{N}1)\}) \geq \Pi 0.7$ we can in fact conclude that $(r,\ \Pi 0.7)$ is entailed by Φ.

In [10] it has been shown that possibilistic resolution is sound and complete in the following sense. Let Φ be a set of possibility- and necessity-valued propositional clauses, or a set of necessity-valued first-order clauses. Then *Incons*$(\Phi) \geq v$ iff there is a derivation of an empty possibilistic clause $(\Box, v)$ from Φ by applications of the possibilistic resolution rule.

Although possibilistic resolution has this nice property, the overall calculus, i.e, transforming arbitrary possibilistic formulas into clause form and then applying the possibilistic resolution rule, has some drawbacks. On the one hand, in the presence of possibility-valued formulas it is in general not possible to transform a set of possibilistic formulas into a set of possibilistic clauses which have the *same* inconsistency degree (see [10], Section 3.1). On the other hand, the calculus applied to a decidable fragment of first-order logic (e.g., terminological logics) may not terminate. But this means that possibilistic resolution in general is neither complete nor terminating even for decidable fragments of first-order logic. Since one goal of the present paper is to instantiate possibilistic logic with some terminological logic this proof method does not yield a decision procedure for a possibilistic extension of terminological logics.

3 An Alternative Proof Method for Possibilistic Logic

This section describes an alternative method for solving the entailment problem in possibilistic logic and for determining the inconsistency degree of a possibilistic knowledge base. The main feature of this method is that it completely abstracts from a concrete calculus, but uses as basic operation a test for classical entailment. If this test is decidable for some fragment of first-order logic, we will see that possibilistic reasoning is also decidable for this fragment. Moreover, if one has an algorithm that solves the entailment problem, our proof method automatically yields an algorithm realizing possibilistic entailment.

In the following we assume that the possibility and necessity degree of a possibilistic formula is not equal to zero. This assumption is justified by the fact that by definition $\Pi(p) \geq 0$ and $\mathrm{N}(p) \geq 0$ hold, which shows that every possibility distribution satisfies formulas of the form $(p, \Pi 0)$ and $(p, \mathrm{N} 0)$, respectively. Hence such formulas do not carry any additional information and can therefore discarded from possibilistic knowledge bases.

Let Φ be a possibilistic knowledge base and let $\alpha \in [0,1]$. We denote by Φ_α (resp. Φ^α) the first-order formulas of necessity-valued formulas in Φ which have a value greater (resp. strictly greater) than α, i.e.,

- $\Phi_\alpha := \{p \mid (p, \mathrm{N}\alpha') \in \Phi,\ \alpha' \geq \alpha\}$ and
- $\Phi^\alpha := \{p \mid (p, \mathrm{N}\alpha') \in \Phi,\ \alpha' > \alpha\}$.

These abbreviations are quite useful to define an alternative characterization of possibilistic entailment. Let Φ be a possibilistic knowledge base, let p be a first-order formula, and let $0 < \alpha \leq 1$. In the following we will show that

- $\Phi \models (p,\ \mathrm{N}\alpha)$ iff $\Phi_\alpha \models p$
- $\Phi \models (p,\ \Pi\alpha)$ iff
  - $\Phi^0 \models p$ or
  - there is some $(q,\ \Pi\beta) \in \Phi$ such that $\beta \geq \alpha$ and $\Phi^{1-\beta} \cup \{q\} \models p$.

This means that $(p,\ \mathrm{N}\,\alpha)$ is entailed by Φ iff the first-order formulas of necessity-valued formulas in Φ whose value is not less than α classically entail p. For possibility-valued formulas the situation is slightly more complex: $(p,\ \Pi\alpha)$ is a possibilistic consequence of Φ iff (1) the first-order formulas of necessity-valued formulas in Φ classically entail p, or (2) there is a possibility-valued formula $(q,\ \Pi\beta)$, $\beta \geq \alpha$, in Φ such that q together with the first-order formulas of necessity-valued formulas in Φ whose value is strictly greater than $1-\beta$ yield a classical proof for p.

We begin with proving soundness of the alternative proof method, i.e. the "if"-part of the above claim.

Lemma 3.1 (Soundness for necessity-valued formulas)
Let Φ be a possibilistic knowledge base and let $(p,\ \mathrm{N}\alpha)$ be a possibilistic formula where $\alpha > 0$. If $\Phi_\alpha \models p$ then $\Phi \models (p,\ \mathrm{N}\alpha)$.

Proof. If $\Phi_\alpha \models p$, there is a subset $\{(p_1,\ \mathrm{N}\,\alpha_1), \ldots, (p_n,\ \mathrm{N}\,\alpha_n)\}$ of Φ such that $\{p_1, \ldots, p_n\} \models p$ and $\alpha \leq \alpha_i$ for all i, $1 \leq i \leq n$. Hence $\alpha \leq \min\{\alpha_1, \ldots, \alpha_n\}$. Let π be a possibility distribution on $\Omega_\perp$ such that $\pi \models \Phi$. We show that π satisfies $(p,\ \mathrm{N}\alpha)$. Observe that $\mathrm{N}(p_i) \geq \alpha_i$ for i, $1 \leq i \leq n$ (because $(p_i,\ \mathrm{N}\alpha_i) \in \Phi$). Since $\{p_1, \ldots, p_n\} \models p$ the formula $p_1 \wedge \ldots \wedge p_n \rightarrow p$ is valid, which shows that

$$\mathrm{N}(p) \geq \mathrm{N}(p_1 \wedge \ldots \wedge p_n) = \min\{\mathrm{N}(p_1), \ldots, \mathrm{N}(p_n)\} \geq \min\{\alpha_1, \ldots, \alpha_n\} \geq \alpha.$$

Thus we have $\mathrm{N}(p) \geq \alpha$ and π satisfies $(p,\ \mathrm{N}\alpha)$. □

Lemma 3.2 (Soundness for possibility-valued formulas)
Let Φ be a possibilistic knowledge base and let $(p,\ \Pi\alpha)$ be a possibilistic formula where $\alpha > 0$. If $\Phi^0 \models p$ or there is some $(q,\ \Pi\beta) \in \Phi$ such that $\beta \geq \alpha$ and $\Phi^{1-\beta} \cup \{q\} \models p$, then $\Phi \models (p,\ \mathrm{N}\alpha)$.

Proof. Assume that $\Phi^0 \models p$. There is a subset $\{(p_1, \mathrm{N}\alpha_1), \ldots, (p_n, \mathrm{N}\alpha_n)\}$ of Φ such that $\{p_1, \ldots, p_n\} \models p$ and $\min\{\alpha_1, \ldots, \alpha_n\} > 0$. This shows that $\mathrm{N}(p) > 0$. Hence we can conclude that for all $\omega \in \Omega_\perp$, $\omega \not\models p$ implies $\pi(\omega) < 1$. Because of the normalization requirement there is an interpretation ω' such that $\pi(\omega') = 1$. Since $\omega' \models p$ it follows that $\Pi(p) = 1$. Thus we can conclude that $\Phi \models (p, \Pi\alpha)$.

Now assume that there is some $(q, \Pi\beta) \in \Phi$ such that $\beta \geq \alpha$ and $\Phi^{1-\beta} \cup \{q\} \models p$. Thus there is a subset $\{(p_1, \mathrm{N}\alpha_1), \ldots, (p_n, \mathrm{N}\alpha_n)\}$ of Φ such that $\{p_1, \ldots, p_n, q\} \models p$ and $\alpha_i > 1 - \beta$ for all i, $1 \leq i \leq n$. Let π be a possibility distribution on $\Omega_\perp$ such that $\pi \models \Phi$. We show that π satisfies $(p, \Pi\alpha)$. Let us recall that

$$\Pi(q) = \max\{\Pi'(q),\ \pi(\omega_\perp)\} \geq \beta \geq \alpha.$$

Case 1: $\Pi(q) = \pi(\omega_\perp)$. Then $\Pi(p) = \sup\{\pi(\omega) \mid \omega \in \Omega_\perp \text{ and } \omega \models p\} \geq \pi(\omega_\perp) \geq \alpha$, which shows that π satisfies $(p, \Pi\alpha)$.
Case 2: $\Pi(q) \neq \pi(\omega_\perp)$. Hence we have $\Pi(q) = \Pi'(q)$. We first show that $\Pi'(q \wedge p_1 \wedge \ldots \wedge p_n) \geq \beta$. Observe that

$$\begin{aligned}
\beta &\leq \Pi'(q) \\
&= \Pi'\big((q \wedge p_1 \wedge \ldots \wedge p_n) \vee (q \wedge \neg(p_1 \wedge \ldots \wedge p_n))\big) \\
&= \Pi'\big((q \wedge p_1 \wedge \ldots \wedge p_n) \vee (q \wedge \neg p_1) \vee \ldots \vee (q \wedge \neg p_n)\big) \\
&= \max\{\Pi'(q \wedge p_1 \wedge \ldots \wedge p_n),\ \Pi'(q \wedge \neg p_1), \ldots,\ \Pi'(q \wedge \neg p_n)\},
\end{aligned}$$

and thus it remains to be shown that $\Pi'(q \wedge \neg p_i) < \beta$ for i, $1 \leq i \leq n$. In fact, since $\mathrm{N}(p_i) = \mathrm{N}'(p_i) \geq \alpha_i$ (which follows from the fact that π satisfies every $(p_i, \mathrm{N}\alpha_i)$) we have $\Pi'(\neg p_i) \leq 1 - \alpha_i$. Recall that $\alpha_i > 1 - \beta$, which shows that $\Pi'(\neg p_i) < \beta$, and therefore $\Pi'(q \wedge \neg p_i) < \beta$ for i, $1 \leq i \leq n$. Thus we can conclude that $\Pi'(q) = \Pi'(q \wedge p_1 \wedge \ldots \wedge p_n) \geq \beta$. Since $\Pi(q \wedge p_1 \wedge \ldots \wedge p_n) \geq \Pi'(q \wedge p_1 \wedge \ldots \wedge p_n)$ and $\{p_1, \ldots, p_n, q\} \models p$ we know that $\Pi(p) \geq \beta \geq \alpha$. Thus π satisfies $(p, \Pi\alpha)$. □

Before we prove completeness of our method we need one more definition and a proposition.

Let Φ be a possibilistic knowledge base containing only necessity-valued formulas. The *canonical possibility distribution*[2] π on $\Omega_\perp$ for Φ is defined by $\pi(\omega) = 1 - \max\{\alpha \mid (p, \mathrm{N}\alpha) \in \Phi \text{ and } \omega \not\models p\}$ where $\max\{\} := 0$.

Proposition 3.3 *Let Φ be a finite set of necessity-valued formulas and let π be the canonical possibility distribution for Φ. Then:*

1. $\pi(\omega) \leq 1 - \alpha$ *if* $(p, \mathrm{N}\alpha) \in \Phi$ *and* $\omega \not\models p$.

[2]Such a distribution is also called *least specific possibility distribution* in [3].

2. π satisfies Φ.

Proof. *1.* Let $(p, \mathrm{N}\alpha) \in \Phi$ and let $\omega \in \Omega_\perp$ be an interpretation such that $\omega \not\models p$. Then $\pi(\omega) = 1 - \max\{\alpha \mid (p, \mathrm{N}\alpha) \in \Phi \text{ and } \omega \not\models p\} \leq 1 - \alpha$.

2. Again, assume that $(p, \mathrm{N}\alpha) \in \Phi$. Since $\pi(\omega) \leq 1 - \alpha$ for every interpretation ω such that $\omega \not\models p$ we have

$$\mathrm{N}(p) = 1 - \sup\{\pi(\omega) \mid \omega \in \Omega_\perp,\ \omega \not\models p\} \geq 1 - (1 - \alpha) = \alpha,$$

which shows that the canonical possibility distribution satisfies every formula in Φ. Furthermore we note that $\pi(\omega_\perp) = 1 - \max\{\alpha \mid (p, \mathrm{N}\alpha) \in \Phi \text{ and } \omega_\perp \not\models p\} = 1$ (because $\omega_\perp \models p$ for all p), which means that the normalization constraint is satisfied. □

Let ω be an interpretation and let Δ be a set of first-order formulas. We say that $\omega \models \Gamma$ iff $\omega \models \gamma$ for each $\gamma \in \Gamma$.

Lemma 3.4 (Completeness for necessity-valued formulas)
Let Φ be a possibilistic knowledge base and let $(p, \mathrm{N}\alpha)$ be a possibilistic formula where $\alpha > 0$. If $\Phi \models (p, \mathrm{N}\alpha)$ then $\Phi_\alpha \models p$.

Proof. Assume that $\Phi \models (p, \mathrm{N}\alpha)$ holds for some $\alpha > 0$. To prove the claim we suppose to the contrary that $\Phi_\alpha \not\models p$ holds. The idea is to construct a possibility distribution π' such that $\pi' \models \Phi \cup \{(\neg p, \mathrm{N}1)\}$ and $\pi'(\omega') > 1 - \alpha$ for some $\omega' \in \Omega$. But this means that *Incons*$(\Phi \cup \{(\neg p, \mathrm{N}1)\}) \leq \mathrm{N}(1 - \pi'(\omega')) < \mathrm{N}\alpha$, which shows that $\Phi \not\models (p, \mathrm{N}\alpha)$ (Proposition 2.3), thus contradicting that $\Phi \models (p, \mathrm{N}\alpha)$ holds.

Let $\Psi := \{(q, \mathrm{N}\beta) \in \Phi \mid \beta \geq \alpha\}$ and let π be the canonical possibility distribution for $\Psi \cup \{(\neg p, \mathrm{N}1)\}$. The possibility distribution π' for $\Phi \cup \{(\neg p, \mathrm{N}1)\}$ is defined as follows:

$$\pi'(\omega) = \begin{cases} \pi(\omega) & \text{if } \omega \not\models \Psi^0 \cup \{\neg p\} \\ 1 & \text{if } \omega = \omega_\perp \\ 1 - {}^1\!/\!_2(\alpha + \gamma) & \text{otherwise,} \end{cases}$$

where $\gamma = \max\{\beta \mid (q, \mathrm{N}\beta) \in \Phi \text{ and } \beta < \alpha\}$. Note that $\gamma < \alpha$, and therefore $1 - {}^1\!/\!_2(\alpha + \gamma) > 1 - \alpha$.

Now we prove that π' satisfies $\Phi \cup \{(\neg p, \mathrm{N}1)\}$, i.e., we show that $\pi' \models \phi$ for all $\phi \in \Phi \cup \{(\neg p, \mathrm{N}1)\}$.

1. $(p', \mathrm{N}\alpha') \in \Psi \cup \{(\neg p, \mathrm{N}1)\}$:
   Then

$$\begin{aligned} \mathrm{N}(p') &= 1 - \sup\{\pi'(\omega) \mid \omega \in \Omega_\perp,\ \omega \not\models p'\} \\ &= 1 - \sup\{\pi(\omega) \mid \omega \in \Omega_\perp,\ \omega \not\models p'\} \qquad \text{(definition of } \pi'\text{)} \\ &\geq \alpha' \qquad \text{(since } \pi \text{ satisfies } (p', \mathrm{N}\alpha')\text{, cf. Proposition 3.3),} \end{aligned}$$

   which shows that $\pi' \models (p', \mathrm{N}\alpha')$.

2. $(p', \text{N}\alpha') \in \Phi \setminus (\Psi \cup \{(\neg p, \text{N}1)\})$:
Consider an interpretation ω' such that $\omega' \not\models p'$. If $\omega' \models \Psi^0 \cup \{\neg p\}$ then $\pi'(\omega') = 1 - 1/2(\alpha + \gamma) < 1 - \gamma$ where, $\gamma = \max\{\beta \mid (q, \text{N}\beta) \in \Phi$ and $\beta < \alpha\}$. Now assume that $\omega' \not\models \Psi^0 \cup \{\neg p\}$. There is some $(p'', \text{N}\alpha'') \in \Psi \cup \{(\neg p, \text{N}1)\}$ such that $\omega \not\models p''$. Since $\pi'(\omega') = \pi(\omega')$ and $\pi(\omega') \leq 1 - \alpha''$ (Proposition 3.3), we can conclude that $\pi'(\omega') \leq 1 - \alpha''$. Since $\alpha'' \geq \alpha > \gamma$ we can conclude that $\pi'(\omega') < 1 - \gamma$.

   Thus we have shown that $\pi'(\omega') < 1 - \gamma$ for every ω' such that $\omega' \not\models p'$. Hence we know that $\text{N}(p') = 1 - \sup\{\pi'(\omega) \mid \omega \in \Omega_\perp,\ \omega \not\models p'\} \geq 1 - (1 - \gamma) = \gamma$. Since $\gamma \geq \alpha'$ (because $(p', \text{N}\alpha') \in \Phi \setminus (\Psi \cup \{(\neg p, \text{N}1)\})$) we can conclude that $\pi' \models (p', \text{N}\alpha')$.

3. $(q, \Pi\beta) \in \Phi$:
Observe that $\Pi(q) = \sup\{\pi'(\omega) \mid \omega \in \Omega_\perp,\ \omega \models q\} \geq \pi'(\omega_\perp) = 1 \geq \beta$, which shows that $\pi' \models (q, \Pi\beta)$.

Thus 1., 2., and 3. together show that $\pi' \models \Phi \cup \{(\neg p, \text{N}1)\}$. Furthermore, we observe that the normalization constraint is obviously satisfied since $\pi(\omega_\perp) = 1$. To complete the proof it remains to be shown that $\pi'(\omega') > 1 - \alpha$ for some $\omega' \in \Omega$.

Recall that we assumed that $\Phi_\alpha \not\models p$. Therefore $\Phi_\alpha \cup \{\neg p\}$ is consistent, which means that there is an interpretation ω' such that $\omega' \models \Phi_\alpha \cup \{\neg p\}$. According to the definition of π' we have $\pi'(\omega') = 1 - 1/2(\alpha + \gamma) > 1 - \alpha$ and we are done. □

Lemma 3.5 (Completeness for possibility-valued formulas)
Let Φ be a possibilistic knowledge base and let $(p, \Pi\alpha)$ be a possibilistic formula where $\alpha > 0$. If $\Phi \models (p, \Pi\alpha)$ then $\Phi^0 \models p$ or there is some $(q, \Pi\beta) \in \Phi$ such that $\beta \geq \alpha$ and $\Phi^{1-\beta} \cup \{q\} \models p$.

Proof. Assume that $\Phi \models (p, \Pi\alpha)$ for some $\alpha > 0$. If $\Phi^0 \models p$ we are done. Thus assume that $\Phi^0 \not\models p$. We show that there is a formula $(q, \Pi\beta)$ in Φ such that $\beta \geq \alpha$ and $\Phi^{1-\beta} \cup \{q\} \models p$.

Suppose to the contrary that for all $(q, \Pi\beta)$ in Φ such that $\beta \geq \alpha$ we have $\Phi^{1-\beta} \cup \{q\} \not\models p$. In the following we construct a possibility distribution π' such that $\pi' \models \Phi \cup \{(\neg p, \text{N}1)\}$ and $\pi'(\omega_\perp) < \alpha$. But this means that $Incons(\Phi \cup \{(\neg p, \text{N}1)\}) < \Pi\alpha$, which shows that $\Phi \not\models (p, \Pi\alpha)$ (Proposition 2.3), thus contradicting that $\Phi \models (p, \Pi\alpha)$ holds.

Let π be the canonical possibility distribution for $\{(p', \text{N}\alpha') \mid (p', \text{N}\alpha') \in \Phi\} \cup \{(\neg p, \text{N}1)\}$. The possibility distribution π' for $\Phi \cup \{(\neg p, \text{N}1)\}$ is constructed as follows:

$$\pi'(\omega) = \begin{cases} \pi(\omega) & \text{if } \omega \not\models \Phi^0 \cup \{\neg p\} \\ 1/2(\alpha + \gamma) & \text{if } \omega = \omega_\perp \\ 1 & \text{otherwise,} \end{cases}$$

where $\gamma = \max\{\beta \mid (r, \mathrm{N}\beta) \in \Phi \text{ and } \beta < \alpha\}$. Observe that $\gamma < \alpha$, which means that $\pi'(\omega_\perp) < \alpha$.

Now we prove that π' satisfies $\Phi \cup \{(\neg p, \mathrm{N}1)\}$, i.e., we show that $\pi' \models \phi$ for every $\phi \in \Phi \cup \{(\neg p, \mathrm{N}1)\}$.

1. $(p', \mathrm{N}\alpha') \in \Phi \cup \{(\neg p, \mathrm{N}1)\}$:
   Then

$$\begin{aligned} \mathrm{N}(p') &= 1 - \sup\{\pi'(\omega) \mid \omega \in \Omega_\perp,\ \omega \not\models p'\} \\ &= 1 - \sup\{\pi(\omega) \mid \omega \in \Omega_\perp,\ \omega \not\models p'\} \qquad \text{(definition of } \pi') \\ &\geq \alpha' \qquad \text{(since } \pi \text{ satisfies } (p', \mathrm{N}\alpha'), \text{ cf. Proposition 3.3),} \end{aligned}$$

   which shows that $\pi' \models (p', \mathrm{N}\alpha')$.

2. $(p', \Pi\alpha') \in \Phi$ where $\alpha' \geq \alpha$.
   Note that $\Pi(p') = \sup\{\pi'(\omega) \mid \omega \in \Omega_\perp,\ \omega \models p'\}$, and it thus suffices to show that there is some $\omega' \in \Omega_\perp$ such that $\omega' \models p'$ and $\pi'(\omega') \geq \alpha'$.

   Case 1: There is an interpretation ω' different from $\omega_\perp$ such that $\omega' \models \Phi^0 \cup \{p', \neg p\}$. Then $\pi'(\omega') = 1$ (definition of π'), which shows that $\Pi(p') \geq \pi'(\omega') = 1 \geq \alpha'$. Thus $\pi' \models (p', \Pi\alpha')$.

   Case 2: Now suppose that $\omega \not\models \Phi^0 \cup \{p', \neg p\}$ for all interpretations ω different from $\omega_\perp$. Recall that we assumed that $\Phi^{1-\alpha'} \cup \{p'\} \not\models p$. This means that $\Phi^{1-\alpha'} \cup \{p', \neg p\}$ is consistent, and hence there is some interpretation ω' such that $\omega' \models \Phi^{1-\alpha'} \cup \{p', \neg p\}$. Since we assumed that $\omega \not\models \Phi^0 \cup \{p', \neg p\}$ for every interpretation ω, we can conclude that there is some $(p'', \mathrm{N}\alpha'') \in \Phi$ such that $\omega' \not\models p''$ and $\alpha'' \leq 1 - \alpha'$.

   Since $\pi'(\omega') = \pi(\omega')$ (definition of π'), it remains to be shown that $\pi(\omega') \geq \alpha'$. In fact,

$$\begin{aligned} \pi(\omega') &= 1 - \max\{\beta \mid (r, \mathrm{N}\beta) \in \Phi \cup \{(\neg p, \mathrm{N}1)\},\ \omega' \not\models r\} \\ &\qquad \text{(definition of } \pi) \\ &= 1 - \max\{\beta \mid (r, \mathrm{N}\beta) \in \Phi \cup \{(\neg p, \mathrm{N}1)\},\ \beta \leq 1 - \alpha',\ \omega' \not\models r\} \\ &\qquad \text{(since } \omega' \models \Phi^{1-\alpha'} \cup \{p', \neg p\}) \\ &\geq \alpha' \qquad \text{(since } \beta \leq 1 - \alpha'). \end{aligned}$$

   Thus we have shown that $\Pi(p') \geq \pi'(\omega') = \pi(\omega') \geq \alpha'$ and therefore we can conclude that $\pi' \models (p', \Pi\alpha')$.

3. $(p', \Pi\alpha') \in \Phi$ where $\alpha' < \alpha$.
Since $\Pi(p') \geq \pi'(\omega_\perp)$ it suffices to show that $\pi'(\omega_\perp) \geq \alpha'$. In fact, $\pi'(\omega_\perp) = {}^1\!/_2(\alpha + \gamma) \geq \alpha'$ (because $\gamma \geq \alpha'$ as well as $\alpha > \alpha'$).

Thus we have proved that π' satisfies $\Phi \cup \{(\neg p,\ \mathrm{N}\,1)\}$. To complete the proof we show that the normalization constraint is satisfied and that $\mathit{Incons}(\Phi \cup \{(\neg p,\ \mathrm{N}1)\}) < \Pi\alpha$. On the one hand, we assumed that $\phi^0 \not\models p$, which means that there is some interpretation ω' such that $\omega' \models \phi^0 \cup \{\neg p\}$. Hence we have $\pi'(\omega') = 1$. On the other hand, we have already mentioned that $\pi'(\omega_\perp) < \alpha$. This, of course, shows $\mathit{Incons}(\Phi \cup \{(\neg p,\ \mathrm{N}1)\}) < \Pi\alpha$. □

The previous lemmas provide us with a proof for the main result of this section.

Theorem 3.6 *Let Φ be a possibilistic knowledge base, let p be a first-order formula, and let $\alpha > 0$. Then*

- $\Phi \models (p,\ \mathrm{N}\alpha)$ *iff* $\Phi_\alpha \models p$ *and*
- $\Phi \models (p,\ \Pi\alpha)$ *iff*
  - $\Phi^0 \models p$ *or*
  - *there is some* $(q,\ \Pi\beta) \in \Phi$ *such that* $\beta \geq \alpha$ *and* $\Phi^{1-\beta} \cup \{q\} \models p$.

Corollary 3.7 *Possibilistic entailment is decidable in those languages in which classical entailment is decidable.*

Let us consider some examples. Assume that Φ is given by $\{(p,\ \mathrm{N}0.8),\ (p \to q,\ \mathrm{N}\,0.4),\ (q \to r,\ \Pi\,0.7)\}$. Since $\{p,\ p \to q,\ q \to r\} \models r$, and $\min\{0.8, 0.4\}{+}0.7 > 1$ we can conclude that Φ entails the possibilistic formula $(r,\ \Pi 0.7)$ (cf. Example 2.1).

Now consider $\Phi' := \Phi \cup \{((q \vee \neg p) \to r,\ \mathrm{N}\,0.5)\}$. Then $\Phi_{0.4} = \{p,\ p \to q,\ (q \vee \neg p) \to r\} \models r$ and $\Phi_{0.5} = \{p,\ (q \vee \neg p) \to r\} \not\models r$, which shows that $\Phi' \models (r,\ \mathrm{N}0.4)$ and $\Phi' \not\models (r,\ \mathrm{N}0.5)$.

The second example in concerned with a possibilistic knowledge base which reveals incompleteness of possibilistic resolution (cf. [10]). Let $\Phi := \{(\forall x\ p(x),\ \Pi\alpha)\}$ for some $\alpha > 0$ and let $p(a) \wedge p(b)$ be a first-order formula. It can easily be verified that possibilistic resolution applied to $\Phi \cup \{(\neg p(a) \vee \neg p(b),\ \mathrm{N}1)\}$ allows one to derive the following three formulas: $(\neg p(a),\ \Pi\alpha)$, $(\neg p(b),\ \Pi\alpha)$, and $(\Box,\ \Pi 0)$. But this means that possibilistic resolution does not recognize that the formula $(p(a) \wedge p(b),\ \Pi\,\alpha)$ is entailed by Φ. In fact, since $\Phi^{1-\alpha} \cup \{\forall x\ p(x)\} = \{\forall x\ p(x)\} \models p(a) \wedge p(b)$, we can conclude that $(p(a) \wedge p(b),\ \Pi\alpha)$ is a possibilistic consequence of Φ.

In the rest of this section we consider the problem of how to determine (with the help of Theorem 3.6) the inconsistency degree of a possibilistic knowledge base Φ. By Proposition 2.3 we know that $\Phi \models (\bot, w)$ iff $Incons(\Phi \cup \{(\neg\bot, \mathrm{N}\,1)\}) \geq w$, and hence $\Phi \models (\bot, w)$ iff $Incons(\Phi) \geq w$, where $\bot$ is a contradiction and w is a necessity or possibility measure. Thus the problem is to find the maximal value w such that $\Phi \models (\bot, w)$, where the total ordering on possibility and necessity measures is defined by $\Pi\alpha \geq \Pi\alpha'$ iff $\alpha \geq \alpha'$, $\mathrm{N}\,\alpha \geq \mathrm{N}\,\alpha'$ iff $\alpha \geq \alpha' > 0$, and $\mathrm{N}\,\alpha \geq \Pi\,\alpha'$ iff $\alpha > 0$ and $\alpha' \leq 1$. This definition is justified by the fact that $Incons(\Phi) \geq \Pi\,\alpha$ (resp. $Incons(\Phi) \geq \mathrm{N}\alpha$) implies $Incons(\Phi) \geq \Pi\alpha'$ (resp. $Incons(\Phi) \geq \mathrm{N}\alpha'$) if $\alpha \geq \alpha'$, and that $Incons(\Phi) \geq \mathrm{N}\,\alpha$ implies $Incons(\Phi) \geq \Pi\alpha'$ if $\alpha > 0$ and $\alpha' \leq 1$.

Let $\gamma := \min\{\alpha \mid (p, \mathrm{N}\alpha) \in \Phi\}$. First assume that $\Phi_\gamma \models \bot$. This means that Φ is necessarily inconsistent at least to degree γ. Observe that $\Phi_\alpha \supseteq \Phi_{\alpha'}$ iff $\alpha \leq \alpha'$. Hence, in order to determine the number $\alpha \in \{\alpha \mid (p, \mathrm{N}\alpha) \in \Phi\}$ such that $\Phi_\alpha \models \bot$ but $\Phi^\alpha \not\models \bot$ one can for instance apply a binary search algorithm (rather than testing for each element $\alpha \in \{\alpha \mid (p, \mathrm{N}\,\alpha) \in \Phi\}$ whether or not Φ_α is inconsistent). The inconsistency degree of Φ is then given by $\mathrm{N}\,\alpha$.

Now assume that $\Phi_\gamma \not\models \bot$. If $\Phi^{1-\beta} \cup \{q\}$ is consistent for every $(q, \Pi\beta)$ in Φ, we can conclude that Φ is completely consistent (which means that $Incons(\Phi) = \Pi 0$). Otherwise, the maximal number β such that $(q, \Pi\beta) \in \Phi$ and $\Phi^{1-\beta} \cup \{q\}$ is inconsistent yields the inconsistency degree of Φ, i.e., $Incons(\Phi) = \Pi\beta$. It should be noted that if $(q, \Pi\beta)$ and $(q', \Pi\beta')$ are in Φ where $\beta \leq \beta'$, in general neither $Th(\{q\} \cup \Phi^{1-\beta}) \subseteq Th(\{q'\} \cup \Phi^{1-\beta'})$ nor $Th(\{q\} \cup \Phi^{1-\beta}) \supseteq Th(\{q'\} \cup \Phi^{1-\beta'})$ hold.[3] This, however, means that one cannot employ binary search to determine the required value β.

These observations show that one can determine with $\mathcal{O}(\log n)$ classical entailment tests the maximal number α such that $\Phi \models (p, \mathrm{N}\,\alpha)$, where Φ is a possibilistic knowledge base containing n formulas and p is a first-order formula. In contrast to this, one can determine with $\mathcal{O}(n)$ entailment tests the maximal number α such that $\Phi \models (p, \Pi\alpha)$.

4 A Possibilistic Extension of Terminological Logics

This section describes an extension of terminological knowledge representation formalisms that handles uncertain knowledge and allows for approximate reasoning. Such an extension—as already argued in [16]—may enhance the expressivity of terminological logics and may thus enlarge their applicability.

[3]Here $Th(\Gamma)$ stands for the deductive closure of a set of first-order formulas Γ.

The idea is to instantiate possibilistic logic with a concrete terminological formalism. This approach is not only satisfactory from a semantical point of view; it also provides us with sound and complete decision procedures for the basic inference problems.

4.1 Terminological knowledge representation

In the following we briefly introduce a particular terminological formalism, called $\mathcal{ALCN}$ (cf. [8]). Such a formalism can be used to define the relevant concepts of a problem domain. Relationships between concepts, for instance inclusion or disjointness axioms, can be expressed in the terminological part. The assertional part allows one to describe objects of the problem domain with respect to their relation to concepts and their interrelation with each other.

Definition 4.1 *We assume two disjoint alphabets of symbols, called* primitive concepts *and* roles*. The set of* concepts *is inductively defined as follows. Every primitive concept is a concept. Now let* C*,* D *be concepts already defined and let* R *be a role. Then* $C \sqcap D$ *(conjunction),* $C \sqcup D$ *(disjunction),* $\neg C$ *(negation),* $\forall R.C$ *(value-restriction),* $\exists R.C$ *(exists-restriction), and* $(\geq n\ R)$ *and* $(\leq n\ R)$ *(number-restrictions) are concepts of the language* $\mathcal{ALCN}$*.*

Concepts are usually interpreted as subsets of a domain and roles as binary relations over a domain. This means that primitive concepts (resp. roles) are considered as symbols for unary (resp. binary) predicates, and that concepts correspond to formulas with one free variable. Thus primitive concepts A and roles R are translated into atomic formulas $A(x)$ and $R(x, y)$ where x, y are free variables. The semantics of the concept-forming constructs is given by

$$
\begin{aligned}
(C \sqcap D)(x) &= C(x) \wedge D(x) \\
(C \sqcup D)(x) &= C(x) \vee D(x) \\
(\neg C)(x) &= \neg C(x) \\
(\forall R.C)(x) &= \forall y\ (R(x,y) \rightarrow C(y)) \\
(\exists R.C)(x) &= \exists y\ (R(x,y) \wedge C(y)) \\
(\geq n\ R)(x) &= \exists y_1, \ldots, y_n\ y_1 \neq y_2 \wedge y_1 \neq y_3 \wedge \ldots \wedge y_{n-1} \neq y_n \\
&\qquad \wedge\ R(x, y_1) \wedge \ldots \wedge R(x, y_n) \\
(\leq n\ R)(x) &:= \forall y_1, \ldots, y_{n+1}\ R(x, y_1) \wedge \ldots \wedge R(x, y_{n+1}) \\
&\qquad \rightarrow y_1 = y_2 \vee y_1 = y_3 \vee \ldots \vee y_{n-1} = y_n
\end{aligned}
$$

It should be noted that the formulas thus obtained belong to a restricted subclass of all first-order formulas with one free variable.

A terminological knowledge base is described by a set of inclusion axioms and—to introduce objects with respect to their relation to concepts and their interrelation with each others—a set of membership assertions.

To be more formally, let C, D be concepts, R be a role, and let a, b be names for individuals, so-called *objects*. A *terminological axiom* is of the form

$$C \rightarrow D,$$

and expresses that every instance of C is also an instance of D. To state that an object a belongs to a concept C, or that two objects a, b are related by a role R one can use *assertions* having the form

$$C(a) \quad \text{or} \quad R(a, b).$$

The semantics of a terminological axiom $C \rightarrow D$ is given by the formula $\forall x\, C(x) \rightarrow D(x)$ where $C(x), D(x)$ are the first-order formulas corresponding to the concepts C, D. To define the semantics of assertions we consider individual names as symbols for constants. In terminological systems one usually has a *unique name assumption*, which can be expressed by the formulas $a \neq b$ for all distinct individual names a, b. The formula corresponding to the assertion $C(a)$ (resp. $R(a, b)$) is obtained by replacing the free variable(s) in the formula corresponding to C (resp. R) by a (resp. a, b).

A *terminological knowledge base* is a pair $(\mathcal{T}, \mathcal{A})$ where $\mathcal{T}$ is a finite set of terminological axioms (the so-called *TBox*) and $\mathcal{A}$ is a finite set of assertions (the so-called *ABox*). Observe that a terminological knowledge base $(\mathcal{T}, \mathcal{A})$ can be viewed as a finite set of first-order formulas that can be obtained by taking the translations of the TBox and ABox facts, and the formulas expressing unique name assumption.

The basic inference services for terminological knowledge bases are defined as follows:

Consistency checking: Does there exist a model for a given terminological knowledge base $(\mathcal{T}, \mathcal{A})$?

Subsumption problem: Is a terminological axiom $C \rightarrow D$ entailed by $(\mathcal{T}, \mathcal{A})$, i.e., $(\mathcal{T}, \mathcal{A}) \models \forall x\, C(x) \rightarrow D(x)$?

Instantiation problem: Is an assertion $C(a)$ (resp. $R(a, b)$) entailed by $(\mathcal{T}, \mathcal{A})$, i.e., $(\mathcal{T}, \mathcal{A}) \models C(a)$ (resp. $(\mathcal{T}, \mathcal{A}) \models R(a, b)$) ?

It should be noted that these inference problems are decidable for most terminological logics.

4.2 The possibilistic extension

The possibilistic extension of the terminological formalism introduced in the previous subsection is obtained as follows: Each terminological axiom (resp. assertion) is equipped with a possibility or a necessity value and will be called *possibilistic terminological axiom* (resp. *possibilistic assertion*). A *possibilistic knowledge base* is now a set of possibilistic terminological axioms together with a set of possibilistic assertions.

In order to give some impression on the expressivity of the extended terminological language let us consider two examples. The first example, which is taken from [16], is concerned with strict terminological axioms but uncertain assertions. Assume that the axioms of $\mathcal{T}$ are given by

$$\begin{array}{rcl} (\mathit{Father} & \leftrightarrow & \mathit{Man} \sqcap (\geq 1\ \mathit{child}),\ \mathrm{N}1) \\ (\mathit{Successful\_father} & \leftrightarrow & \mathit{Father} \sqcap \forall \mathit{child}.\mathit{College\_graduate},\ \mathrm{N}1), \end{array}$$

where $(C \leftrightarrow D,\ \mathrm{N}1)$ is an abbreviation for $(C \rightarrow D,\ \mathrm{N}1)$ and $(D \rightarrow C,\ \mathrm{N}1)$. The first axiom expresses that someone is a father iff he is a man and has some child; the latter one states that someone is a successful father iff he is a father and all his children are college graduates.

First consider the (certain) assertions

$$\begin{array}{rl} \mathcal{A} = \{ & (\mathit{Man} \sqcap (\leq 2\ \mathit{child})(\mathit{John}),\ \mathrm{N}1), \\ & (\mathit{child}(\mathit{John}, \mathit{Philip}),\ \mathrm{N}1), \\ & (\mathit{child}(\mathit{John}, \mathit{Angela}),\ \mathrm{N}1), \\ & (\mathit{College\_graduate}(\mathit{Philip}),\ \mathrm{N}1), \\ & (\mathit{College\_graduate}(\mathit{Angela}),\ \mathrm{N}1)\ \}, \end{array}$$

which state that John is a man having at most two children, that Philip and Angela are children of John, and that both are college graduates. Since Philip and Angela are the only children of John (because he has at most two children) and both children are college graduates, we can conclude that John is a successful father, i.e., $(\mathit{Successful\_father}(\mathit{John}),\ \mathrm{N}1)$ is entailed by $(\mathcal{T}, \mathcal{A})$. Now assume that it is only likely (but not sure) that Philip is a college graduate, which can be expressed by $(\mathit{College\_graduate}(\mathit{Philip}),\ \mathrm{N}0.8)$. Then possibilistic entailment allows one to conclude that John is a successful father but, of course, only with a necessity degree of 0.8.

In the second example possibility and necessity degrees are utilized to express plausible rules. Assume that the terminology $\mathcal{T}$ consists of the following possibilistic axioms:

$$\begin{array}{rcl} (\exists \mathit{owns}.\mathit{Porsche} & \rightarrow & \mathit{Rich\_person} \sqcup \mathit{Car\_fanatic},\ \mathrm{N}0.8) \\ (\mathit{Rich\_person} & \rightarrow & \mathit{Golfer},\ \Pi 0.7). \end{array}$$

The first rule expresses that it is rather certain that someone is either rich or a car fanatic if (s)he owns a Porsche. The second rule says that rich persons are possibly golfers. The assertional knowledge is given by the facts that Tom owns a Porsche 911 and that he is probably not a car fanatic, i.e., $\mathcal{A}$ has the form

$$\{(Owns(Tom, 911), \text{N}1), (Porsche(911), \text{N}1), (\neg Car\_fanatic(Tom), \text{N}0.7)\}.$$

We are interested in the question of whether or not that Tom is a golfer. To answer the question observe that

$$\{Owns(Tom, 911), Porsche(911)\} \models (\exists owns.Porsche)(Tom),$$

which shows that $(\mathcal{T}, \mathcal{A})$ entails $((\exists owns.Porsche)(Tom), \text{N}1)$. Hence, it can easily be verified that $(\mathcal{T}, \mathcal{A})^{1-0.7} \cup \{Rich\_person(Tom)\} \models Golfer(Tom)$. This shows that $(Golfer(Tom), \Pi 0.7)$ is a possibilistic consequence of $(\mathcal{T}, \mathcal{A})$, which means that we have some reasons to believe that Tom is a golfer.

The following proposition shows that possibilistic reasoning restricted to the introduced terminological formalism is decidable. This result is an immediate consequence of Theorem 3.6 and the fact that the instantiation problem in $\mathcal{ALCN}$-knowledge bases is decidable (cf. [5]).

Proposition 4.2 *Let $\mathcal{T}$ be a finite set of possibilistic axioms and let $\mathcal{A}$ be a finite set of possibilistic assertions. If ϕ is a possibilistic axiom or a possibilistic assertion the question of whether or not ϕ is entailed by $(\mathcal{T}, \mathcal{A})$ is decidable.*

5 Conclusion

We have developed an alternative proof method for possibilistic logic which exploits the fact that possibilistic reasoning can be reduced to reasoning in classical, i.e. first-order, logic. Consequently, possibilistic reasoning is decidable for a fragment of first-order logic iff classical entailment is decidable for it. Moreover, if one has an algorithm solving the entailment problem, our method automatically yields an algorithm realizing possibilistic entailment which is sound and complete with respect to the semantics for possibilistic logic.

Furthermore, we have instantiated possibilistic logic with a terminological logic, which is a decidable fragment of first-order logic but nevertheless much more expressive than propositional logic. This leads to an extension of terminological logics towards the representation of uncertain knowledge which is—in contrast to other approaches—satisfactory from a semantic point of view. Moreover, a sound and complete algorithm for possibilistic entailment in such an extension can be obtained by using inference procedures which have already been developed for terminological logics.

An interesting point for further research is to employ possibilistic logic in order to represent and reason with defaults in terminological formalisms. In fact, in [6, 3] it has been argued that possibilistic logic yields a good basis for nonmonotonic reasoning. Roughly speaking, the idea is as follows: If the necessity of a formula p is greater than the necessity of $\neg p$ with respect to a set Φ of necessity-valued formulas, then infer nonmonotonically p from Φ. That this intuitive definition in fact characterizes an appropriate nonmonotonic consequence relation is partially justified by the facts that (1) the operator can be described in terms of preferential models, and that (2) most of the axioms which a nonmonotonic operator should satisfy are met. (see [6] on these points). The approach presented in [3], however, uses propositional logic and cannot directly be applied to the terminological case. One reason for this is that terminological default rules usually allow one to state that "C's are normally D's" where C, D are concepts, i.e., first-order formulas with one *free* variable.

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