Rapid Qualification of Mereotopological Relationships using Signed Distance Fields

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***Abstract*—Although mereotopological relationship theories and their qualification problems have been extensively studied in $\mathbb{R}^2$, the qualification of mereotopological relations in $\mathbb{R}^3$ remains challenging. This is due to the limited availability of topological operators and high costs of boundary intersection tests. In this paper, a novel qualification technique for mereotopological relations in $\mathbb{R}^3$ is presented. Our technique rapidly computes RCC-8 base relations using precomputed signed distance fields, and makes no assumptions with regards to complexity or representation method of the spatial entities under consideration.**

I. Introduction

Qualitative spatial reasoning (QSR) is concerned with knowledge representations of spatial entities and their configurations as well as with reasoning and query processing mechanisms for such spatial configurations. It has applications in areas such as robotic navigation [1], scene understanding [2] or geographic information systems [3]. During the last two decades a plethora of QSR calculi has been proposed, perhaps the most well known being the region connection calculus (RCC), a theory which uses an axiomatic framework to describe spatial entities by their pairwise mereotopological relations. Howsoever and regardless of the peculiarities of its diverse calculi, QSR research generally addresses the following problems:

- **P.1** How to qualitatively represent spatial entities and their relationships?
- **P.2** How to infer new relationships from a finite set of qualitative spatial relationships.
- **P.3** What kind of techniques to apply for detecting qualitative from quantitative spatial relationships.

In the scope of this paper, we are concerned with the problem of *Qualification* (**P.3**) and wish to automatically turn quantitative descriptions *of three-dimensional spatial configurations* into qualitative descriptions composed of RCC-8 base relations. For this purpose, a novel qualification technique is presented. Our approach rapidly computes RCC-8 base relations using signed distance fields as lookup data structures, and it has three main advantages:

- **A.1** No assumption is made with regards to the 3D representation techniques of spatial entities.
- **A.2** Complex spatial entities and configurations can be represented to any specified resolution.
- **A.3** Determining the intersection of the boundaries, interiors, and exteriors of two spatial entities is fast and independent of the complexity of the spatial entities.

(a) *Joe's Ice Cream*

(b) *Armadillos* [4]

Fig. 1: RCC-8 qualification of (1a) took 1.051 ms and resulted in *externallyConnected(Joe,Ice)*, *disconnected(Joe,Bucket)*, *disconnected(Ice,Bucket)*. RCC-8 qualification of (1b) took 7.813 ms and resulted in *partiallyOverlapping(A,B)*.

The remainder of this paper is organised as follows. Section II introduces some preliminaries on mereotopological relationship theories and discusses the related literature on mereotopological qualification of spatial configurations in $\mathbb{R}^3$. Section III explains the concept of signed distance fields. We propose our novel qualification algorithm using signed distance fields in Section IV, and report on implementation and performance results in Section V. We conclude with summary in Section VI.

II. Mereotopological Relationship Theories

As a framework for the definition of mereotopological relations between spatial entities embedded in $\mathbb{R}^n$, some prominent QSR theories specify a finite number of jointly exhaustive and pairwise disjoint (JEPD) binary relations in terms of the topological operators *interior* $\cdot^o$, *boundary* $\partial\cdot$ and *exterior* $\cdot^e$ along with the intersection operation from set theory [5].

For example, the 9-Intersection Model (9IM) [6] defines the possible relations between two spatial entities $A, B \subset \mathbb{R}^n$ in terms of the intersections of A's boundary (∂A), interior (A^o) and exterior (A^e) with the boundary (∂B), interior (B^o) and exterior (B^e) of B.

$$\mathrm{9IM}(A,B) = \begin{bmatrix} A^o \cap B^o & A^o \cap \partial B & A^o \cap B^e \\ \partial A \cap B^o & \partial A \cap \partial B & \partial A \cap B^e \\ A^e \cap B^o & A^e \cap \partial B & A^e \cap B^e \end{bmatrix}$$

The existence of intersections is registered in $\mathrm{9IM}(A,B) \in \mathbb{B}^{3\times 3}$, a matrix that precisely describes the topological relation between A and B.

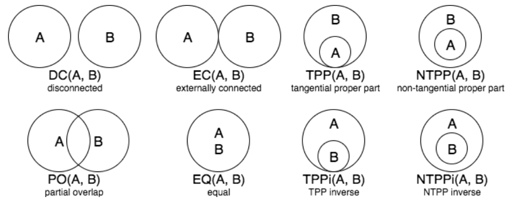


Fig. 2: The eight basic relations of the RCC-8 calculus.

	$A^o \cap \partial B$	$A^e \cap \partial B$	$\partial A \cap \partial B$	$B^o \cap \partial A$
NTPP(A, B)	1	0	0	0
NTPPi(A, B)	0	1	0	1
DC(A, B)	0	1	0	0
TPP(A, B)	1	0	1	0
TPPi(A, B)	0	1	1	1
EC(A, B)	0	1	1	0
PO(A, B)	1	1	1	1
EQ(A, B)	0	0	1	0

TABLE I: Basic relations of RCC-8 in terms of 9IM.

Based on 9IM(A, B), eight qualitative spatial relations between A and B can be distinguished [7]. These relations (cf. Table I) are equivalent to the relations defined by the RCC-8 calculus [8], a first-order logical language for formalising topological relationships between abstract spatial regions (cf. Figure 2).

Although mereotopological relationship theories, e.g. RCC-8 and 9IM, and their qualification [9] have been extensively studied in $\mathbb{R}^2$, the qualification of mereotopological relations in $\mathbb{R}^3$ remains challenging. This is due to

C.1 the limited availability of topological operators interior $\cdot^o$, boundary $\partial\cdot$ and exterior $\cdot^e$, and
C.2 the computational costs of the boundary intersection tests $A^o \cap \partial B$, $A^e \cap \partial B$, $\partial A \cap \partial B$ and $B^o \cap \partial A$

with respect to a specific representation technique for three-dimensional spatial entities. In what follows, we briefly discuss the related literature in this respect.

Surface Models. Albath et al. introduce RCC-3D [10], [11], a QSR system based on the Generalised 2D Region Connection Calculus. In order to determine the mereotopological relation between two 3D spatial entities, RCC-3D performs pairwise triangle-triangle intersections in worst-case time complexity of $O(n^5)$ with n being the maximum number of faces or vertices used to define the spatial domains under consideration. Sabharwal et al. [12]–[17] present VRCC-3D+, an extension of RCC-3D that relies on a decision framework for more efficient calculations of boundary intersections using either triangle-triangle intersection tests or intersection tests with AABB trees.

Solid Modelling. Borrmann et al. [18] introduce a QSR system for the spatial analysis of 3D CAD models. Spatial domains are individually encoded to any specified resolution in octrees with each octree node specifying if it lies completely inside, outside or on the boundary of the encoded spatial domain. Mereotopological qualification between two given spatial domains is performed by synchronised breadth-first traversals of both octrees and the application of 21 decision rules on each traversal level. Ben Hmida et al. [19], [20] present a method to automatically compute 9IM topological relations between Nef polyhedra. The procedure is based on the generation of Selective Nef Complexes (SNC), a representation of Nef polyhedra providing binary Boolean operators and unary operators such as interior, closure and boundary, from standard polyhedra, and the redefinition of the 9IM matrix using solely SNC operators.

III. Signed Distance Fields in brief

Distance fields are a popular data structure in the fields of computer graphics, geometric modelling and robotics and serve a vast range of applications including surface reconstruction [21], shape representation [22] or collision detection [23].

Given a three-dimensional spatial entity $\mathcal{P} \subset \mathbb{R}^3$ with distance metric $d : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$, we denote with $\partial\mathcal{P} \subset \mathcal{P}$ the surface (or boundary) of $\mathcal{P}$. A *distance field* of $\mathcal{P}$ is a scalar field $D' : \mathbb{R}^3 \to \mathbb{R}$ that specifies the minimum distance from any $\boldsymbol{q} \in \mathbb{R}^3$ to the closest point $\boldsymbol{p} \in \partial\mathcal{P}$ such that $D'(\boldsymbol{q}) = min_{\boldsymbol{p} \in \partial\mathcal{P}} \{d(\boldsymbol{p}, \boldsymbol{q})\}$. If $\partial\mathcal{P}$ is a closed surface, i.e., $\partial\mathcal{P}$ is compact and without boundary, we can define a sign function $\sigma' : \mathbb{R}^3 \to \{-1, 1\}$ such that

$$\sigma'(\boldsymbol{q}) = \begin{cases} -1 & \text{if } \boldsymbol{q} \in \mathcal{P} \\ 1 & \text{if } \boldsymbol{q} \notin \mathcal{P} \end{cases}$$

for any $\boldsymbol{q} \in \mathbb{R}^3$. If $\partial\mathcal{P}$ is oriented, i.e., a face normal $\boldsymbol{n} \in \mathbb{R}^3$ is known for every $\boldsymbol{p} \in \partial\mathcal{P}$, we can define a sign function $\sigma : \mathbb{R}^3 \to \{-1, 1\}$ such that

$$\sigma(\boldsymbol{q}) = \begin{cases} -1 & \text{if } (\boldsymbol{q} - \boldsymbol{p}) \cdot \boldsymbol{n} < 0 \\ 1 & \text{if } (\boldsymbol{q} - \boldsymbol{p}) \cdot \boldsymbol{n} \geq 0 \end{cases}$$

As noted by Xu et al. [24], we can find for sufficiently small $\Delta \in \mathbb{R}$ a point $\boldsymbol{q} \in \mathbb{R}^3$ for every $\boldsymbol{p} \in \partial\mathcal{P}$ within the offset surface $\partial_\Delta\mathcal{P} = \left\{\boldsymbol{q} \in \mathbb{R}^3 \| D'(\boldsymbol{q}) = \Delta\right\}$ such that $\sigma(\boldsymbol{q}) = \sigma'(\boldsymbol{q})$. Hence, we can define a *signed distance field* $D : \mathbb{R}^3 \to \mathbb{R}$ with $D(\boldsymbol{q}) = \sigma(\boldsymbol{q}) \cdot D'(\boldsymbol{q})$ for any oriented, not necessarily boundary-free, three-dimensional spatial entity $\mathcal{P} \subset \mathbb{R}^3$.

The problem of computing a distance field is fairly well understood, and there is a wide array of algorithms for triangle meshes, triangle soups, implicit surfaces and parametric surfaces [25].

IV. Qualification using Signed Distance Fields

Given two spatial entities $\mathcal{P}_1, \mathcal{P}_2 \subset \mathbb{R}^3$ with body-fixed coordinate systems $\mathcal{C}_1$ and $\mathcal{C}_2$ and a 6 degree-of-freedom transformation $\boldsymbol{P}$ between $\mathcal{C}_1$ and $\mathcal{C}_2$, our objective is to turn this quantitative geometric description into a qualitative description composed of RCC-8 base relations.

We propose to solve this qualification problem using precomputed signed distance fields as lookup data structures for boundary intersection tests.

Algorithm 1 Histogram computation

```
procedure BUILDHISTOGRAM(𝒫₁, 𝒫₂, P)
    B∂, Bo, Be ← 0
    for all x ∈ ∂𝒫₂ do
        B∂ ← B∂ + 1∂(𝒫₁, Px)
        Bo ← Bo + 1o(𝒫₁, Px)
        Be ← Be + 1e(𝒫₁, Px)
    end for
    return B∂, Bo, Be
end procedure
```

Algorithm 2 RCC-8 qualification algorithm

```
procedure QUALIFY(𝒫₁, 𝒫₂, P)
    B∂, Bo, Be ← BUILDHISTOGRAM(𝒫₁, 𝒫₂, P)
    if B∂ = Be = 0 then
        return NTPP(𝒫₁, 𝒫₂)
    else if Bo = Be = 0 then
        return EQ(𝒫₁, 𝒫₂)
    else if Be = 0 then
        return TPP(𝒫₁, 𝒫₂)
    else if B∂ = Bo = 0 then
        ·, Bo, · ← BUILDHISTOGRAM(𝒫₂, 𝒫₁, P)
        if Bo = 0 then
            return DC(𝒫₁, 𝒫₂)
        else
            return NPTPi(𝒫₁, 𝒫₂)
        end if
    else if Bo = 0 then
        ·, Bo, · ← BUILDHISTOGRAM(𝒫₂, 𝒫₁, P)
        if Bo = 0 then
            return EC(𝒫₁, 𝒫₂)
        else
            return TPPi(𝒫₁, 𝒫₂)
        end if
    else
        return PO(𝒫₁, 𝒫₂)
    end if
end procedure
```

For both spatial entities $\mathcal{P}_1$ and $\mathcal{P}_2$, we assume the availability of

R.1 signed distance fields $D_{\mathcal{P}_1}$ and $D_{\mathcal{P}_1}$ precomputed with one of the methods mentioned in Section III, as well as

R.2 point-based representations of boundaries $\partial\mathcal{P}_1$ and $\partial\mathcal{P}_2$ obtained either directly from three-dimensional polygonal models or by point sampling [26] of $\mathcal{P}_1$ and $\mathcal{P}_2$.

We consider assumptions **R.1** and **R.2** to be justified for non-deformable spatial entities $\mathcal{P}_1$ and $\mathcal{P}_2$, and, therefore, in conformity with related work.

Our qualification technique relies on the fast computation of histograms specified by the following indicator functions

$$\mathbf{1}_\partial(\mathcal{P}_i, \boldsymbol{x}) = \begin{cases} 1 & \text{if } D_{\mathcal{P}_i}(\boldsymbol{x}) = 0 \\ 0 & \text{otherwise} \end{cases} \qquad \mathbf{1}_o(\mathcal{P}_i, \boldsymbol{x}) = \begin{cases} 1 & \text{if } D_{\mathcal{P}_i}(\boldsymbol{x}) < 0 \\ 0 & \text{otherwise} \end{cases}$$

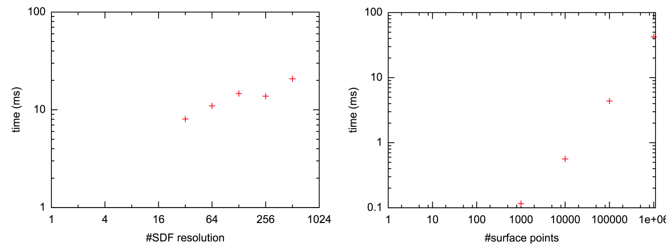
$$\mathbf{1}_e(\mathcal{P}_i, \boldsymbol{x}) = \begin{cases} 1 & \text{if } D_{\mathcal{P}_i}(\boldsymbol{x}) > 0 \\ 0 & \text{otherwise} \end{cases}$$

The indicator functions $\mathbf{1}_\partial(\mathcal{P}_i, \boldsymbol{x})$, $\mathbf{1}_o(\mathcal{P}_i, \boldsymbol{x})$ and $\mathbf{1}_e(\mathcal{P}_i, \boldsymbol{x})$ employ a precomputed signed distance field $D_{\mathcal{P}_i}$ in order to determine if a given point $\boldsymbol{x} \in \mathbb{R}^3$ lies completely inside, outside or on the boundary of the spatial domain $\mathcal{P}_i$.

BUILDHISTOGRAM (cf. Algorithm 1) effectively computes the boundary intersection tests $\mathcal{P}_1^o \cap \partial\mathcal{P}_2$, $\mathcal{P}_1^e \cap \partial\mathcal{P}_2$ and $\partial\mathcal{P}_1 \cap \partial\mathcal{P}_2$ in time complexity of $O(n)$ with n being the number of points in $\partial\mathcal{P}_2$. Further performance optimisations are achievable by exploiting obvious loop-level parallelism.

A	B	#points	qualify(A,B)	Time (ms)
Joe	Ice	6,301	EC(A,B)	0.821
Joe	Bucket	6,530	DC(A,B)	0.184
Ice	Bucket	723	DC(A,B)	0.046
Armadillo 1	Armadillo 2	172,974	PO(A,B)	7.813

TABLE II: Duration of QUALIFY(A, B) for given models with SDF of resolution 32x32x32.



(a) (b)

Fig. 3: Scaling behaviour of QUALIFY(A, B) with respect to (3a) varying SDF resolutions, and (3b) point-based boundary representations of increasing sizes for Armadillos configuration.

Our overall qualification technique QUALIFY (cf. Algorithm 2) relies on BUILDHISTOGRAM in order to perform the boundary intersection tests $\mathcal{P}_1^o \cap \partial\mathcal{P}_2$, $\mathcal{P}_1^e \cap \partial\mathcal{P}_2$, $\partial\mathcal{P}_1 \cap \partial\mathcal{P}_2$ and $\mathcal{P}_2^o \cap \partial\mathcal{P}_1$. Essentially, QUALIFY is a conditional cascade of Table I and rather self-explanatory.

V. IMPLEMENTATION AND RESULTS

The implementation of our approach in C++ relies on the Vega FEM Library [27] for precomputing a signed distance field for each spatial entity. Furthermore, we utilise a Meshlab server [28] for precomputing a point-based surface representation for each spatial entity. All timings reported in this paper are generated on a MacBook Pro 2,8 GHz Intel Core i7 with 16 GB 1600 MHz DDR3 and an NVIDIA GeForce GT 750M 2048 MB, running MacOS Sierra 10.12.6.

We have applied our approach to the following spatial configurations and measured the duration of computing qualitative descriptions composed of basic RCC-8 relations. All timings (in milliseconds) are presented in Table II.

Joe's Ice Cream (cf. Figure 1a) is composed of three spatial entities, i.e., *Joe* (6054 vertices, 12988 faces, 16 connected components, two-manifold, 6 holes, genus 9), *Ice Lolly* (247 vertices, 436 faces, 3 connected components, two-manifold, 3 holes, genus 0), and *Bucket* (476 vertices, 896 faces, 3 connected components, two-manifold, 2 holes, genus 0). RCC-8 qualification using our approach correctly results in EC(Joe,Ice), DC(Joe,Bucket) and DC(Ice,Bucket).

Armadillos (cf. Figure 1b) is composed of two overlapping instances of the *Stanford Armadillo* [4] (172974 vertices, 345944 faces, 1 connected component, two-manifold, 0 holes, genus 0). RCC-8 qualification using our approach correctly results in PO(A,B).

Briefly looking into the scaling behaviour our aproach, we qualified the Armadillos configuration with varying SDF resolutions and fixed-sized point-based boundary representations (cf. Figure 3a) as well as with point-based boundary representations of increasing sizes and a fixed SDF resolution of 32x32x32 (cf. Figure 3b). We discovered the scaling behaviour of our approach to be nearly invariant with respect to the resolution of distance fields and to be linear in the size of point-based boundary representations.

VI. Conlusions

In this paper, we propose a novel qualification technique for mereotopological relations in $\mathbb{R}^3$. Our approach rapidly computes RCC-8 base relations using precomputed signed distance fields, and makes no assumptions with regards to complexity or 3D representation method of the spatial entities under consideration. Conducted performance evaluations suggest improvements over relevant related works as well as favourable scaling behaviour.

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